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Practical Packing Method in Somewhat Homomorphic Encryption

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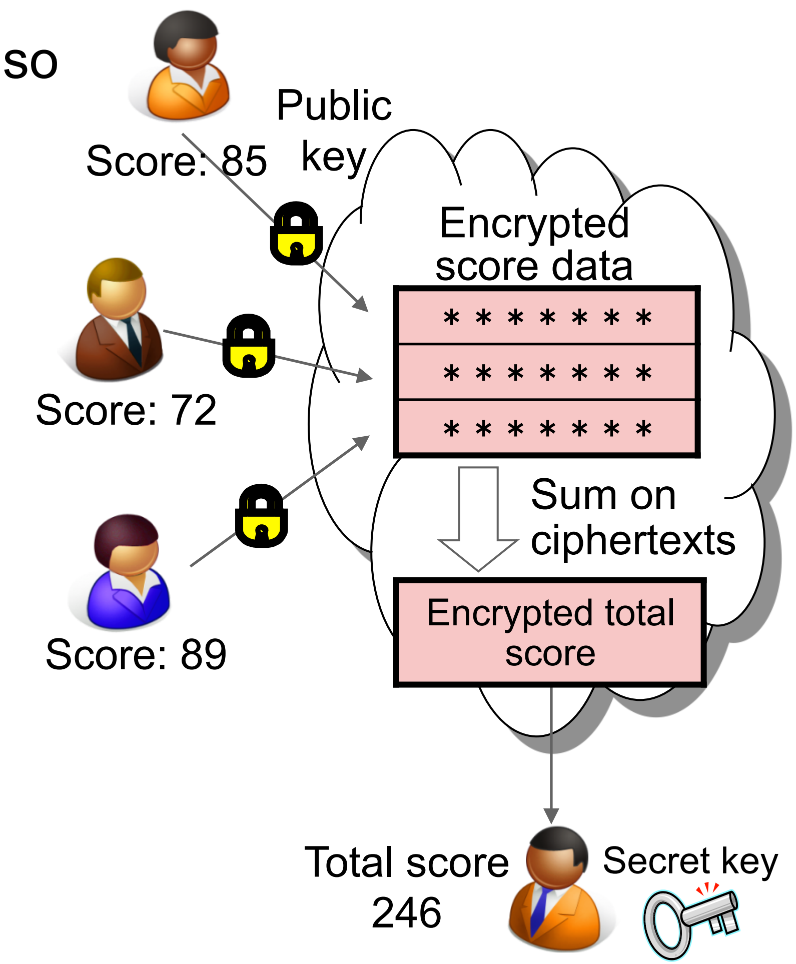
Homomorphic Encryption (HE)

- Public-key encryption supporting "some operations" on encrypted data
  - It enables us to compute the total score so that the cloud cannot know any score
- Additively HE
  - Practical in performance, but it can only support addition (e.g., Paillier scheme)
  - Limited applications (e.g., e-voting)
- FHE (fully HE)
  - Breakthrough by Gentry [1] in 2009
  - It supports any operations, and is expected to applied to cloud computing
  - But, difficulty on performance and size



[1] C. Gentry, "Fully homomorphic encryption using ideal lattices", STOC 2009.

Summary of Our Work

- SHE (Somewhat HE)
  - It supports “a limited number” of additions and multiplications
    - Well known as a building block for the FHE construction
  - Much faster and shorter than FHE
    - (Performance and Size) Additively HE < SHE << FHE
- Motivation: practical use of SHE in wider applications
- Contributions:
  - New packing method in SHE for the practical use
    - Use of the ring-LWE based LWE scheme
    - Our method can pack a vector into a single ciphertexts
    - It gives several efficient computations over packed ciphertexts
  - Application to privacy-preserving biometrics
    - Our method gives faster performance and shorter size for secret authentication in the HE approach

Ring-LWE Based SHE Scheme

Key Generation

- Secret key $\mathrm{sk}=s$, Public key $\mathrm{pk}=(a_0, a_1)$
  - $a_0=-(a_1 s+te)$ in R_q, s, a_1, e: small noises in R_q
  - $R=\mathbb{Z}[x]/(x^n+1)$: base ring, t: plaintext modulus, q: ciphertext modulus

Encryption

- For a palintext $m\in R_t$, $\mathrm{Enc}(m,\mathrm{pk})=(c_0, c_1)\in(R_q)^2$
  - $c_0=a_0 u+tg+m$, $c_1=a_1 u+tf$
  - u, g, f: small noises in R_q

Homomorphic Operations

- [Add] $\mathrm{Enc}(m,\mathrm{pk})+\mathrm{Enc}(m', \mathrm{pk}):=(c_0+c'_0, c_1+c'_1)$
- [Mul] $\mathrm{Enc}(m,\mathrm{pk})*\mathrm{Enc}(m', \mathrm{pk}):=(c_0\cdot c'_0, c_0\cdot c'_1+c'_0\cdot c_1, c_1\cdot c'_1)$

Decryption

- For a ciphertext $\mathrm{ct}=(c_0, c_1, \ldots, c_k)$, $\mathrm{Dec}(\mathrm{ct},\mathrm{sk})=[\sum c_i\cdot s^i]_q \bmod t$ in R_t
  - $[a]_q$: a modulo q in $[-q/2, q/2)$

Introduction of Our Packing Method

Strategy

1. Transform a vector of length n to a certain polynomial in R
2. Pack its polynomial into a single ciphertext

Our Trick: two types of polynomials in $R = \mathbb{Z}[x]/(x^n + 1)$

- [Type1] $A = (A_0, \cdots, A_{n-1}) \rightarrow F_1(A) := \sum A_i x^i$ (ascending order)
- [Type2] $B = (B_0, \cdots, B_{n-1}) \rightarrow F_2(B) := -\sum B_i x^{n-i}$ (descending order)

Packed ciphertexts

- [Type1] $\mathrm{vEnc}_1(A) := \mathrm{Enc}(F_1(A), \mathrm{pk})$
- [Type2] $\mathrm{vEnc}_2(B) := \mathrm{Enc}(F_2(B), \mathrm{pk})$
  - It does not change the security level of the SHE scheme

Efficient Secure Inner Product

Only one homomorphic multiplication

$$\mathrm{vEnc}_1(A) * \mathrm{vEnc}_2(B)$$

over packed ciphertexts gives the inner product between A and B on encrypted data

Sketch of Proof)

$F_1(A) \times F_2(B)$ over plaintexts ←corresponding→ $\mathrm{vEnc}_1(A) * \mathrm{vEnc}_2(B)$

performed on encrypted data

$$F_1(A) = A_0 + A_1 x + A_2 x^2 + \cdots + A_{n-1} x^{n-1}$$

$$\times)\quad F_2(B) = -B_{n-1} x - \cdots - B_1 x^{n-1} - B_0 x^n$$

$$F_1(A) \times F_2(B) = -(A_0 B_0 + A_1 B_1 + \cdots A_{n-1} B_{n-1}) x^n + \cdots$$

$x^n = -1$ in R

The constant term gives our desired inner product

Computations over Packed Ciphertexts

Combinations of our packed ciphertexts also give us the following computations:

- Private Statistic:
  - Sum, and Mean
  - Variance, and Standard deviation
- Statistical Analysis:
  - Covariance
  - Correlation
- Distances
  - Hamming distance
  - Euclid distance
- etc.,…

Remark: It is difficult to perform computations such as
- the median,
- the 1-norm distance

since we generally can not compare two values without decryption in the use of homomorphic encryption (irrespective of packing method).

Application to Privacy-Preserving Biometrics

Privacy-Preserving Biometrics

- Biometric authentication with protecting the privacy of biometric data

Homomorphic Encryption Approach

- **Secure Hamming distance**: metric to measure the similarity of feature vectors A, B on encrypted data
  - $d_H(A,B)=\sum_{i=0}^{2047}(A_i-B_i)^2 = \sum_{i=0}^{2047}(A_i+B_i-2A_i\cdot B_i)$
  - The authentication result is "OK" if $d_H(A,B)<\sigma$

Biometric Image

010101011
101001111
001100101
110101010

Feature Vector

Homomorphic Encryption

412ABCD11
124913FA12
FF121AB123
9192BBA112

Assume 2048-bit for various biometrics (e.g., vein codes)

Secure Hamming distance in authentication

Secret Authentication with Our Method

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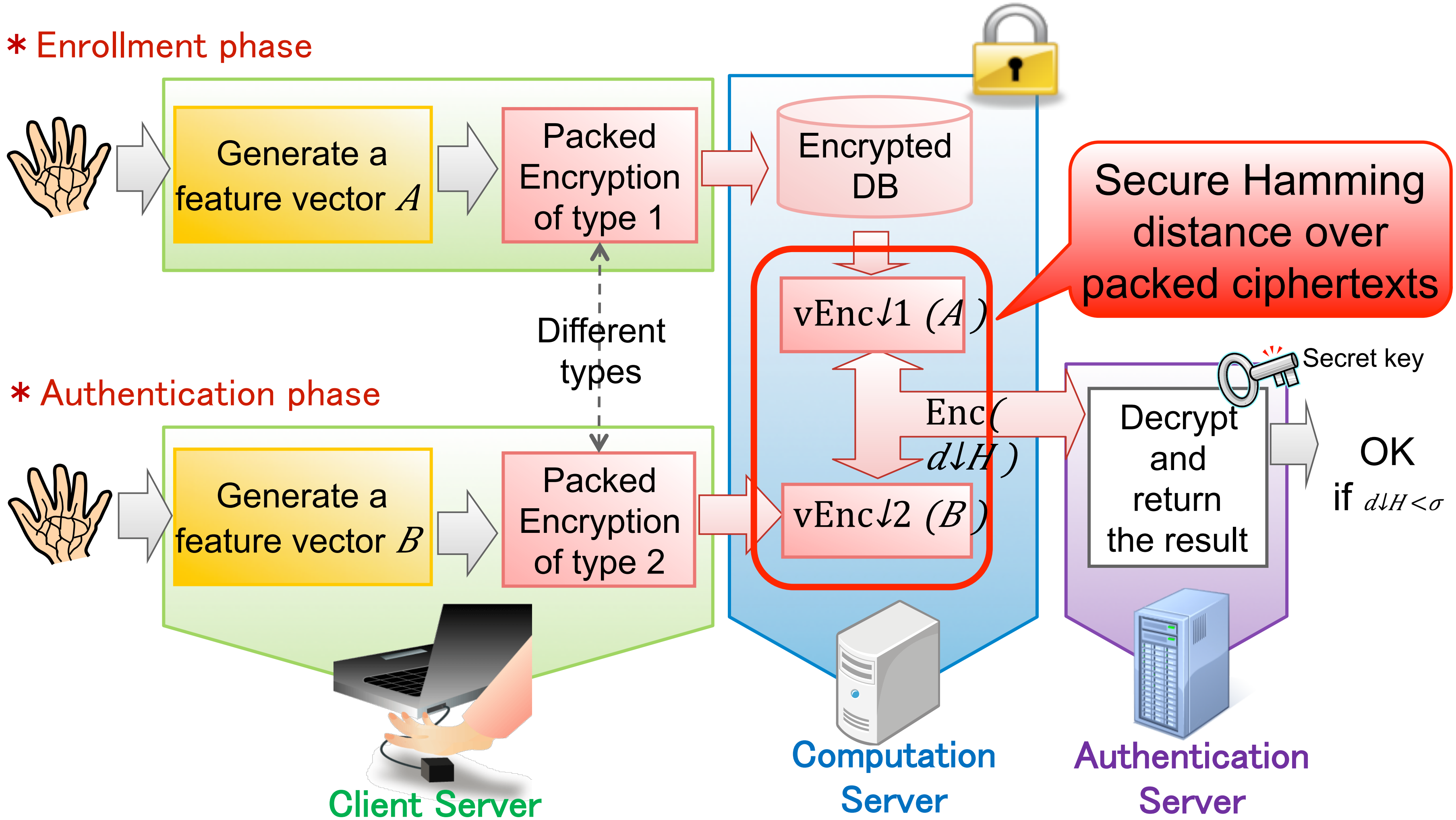
- Since all computations are performed on encrypted data, we hope that we would use “the cloud” as the computation server



Comparison with Related Work

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Table: A comparison on the performance and the encrypted data size

Protocols (feature vector size)	Performance of Secure Hamming	Size increase rate by encryption† (cipher. size)	Homomorphic encryption scheme
SCiFI [25] (900-bit)	310 ms[(a)]	2048 times (230 KByte)	Paillier-1024 (additive scheme)
Protocol of [2] (2048-bit)	150 ms[(b)]	1024 times (262 KByte)	DGK-1024 (additive scheme)
Previous work‡ [31] (2048-bit)	18.10 ms[(c)]	about 80 times (19 KByte)	ideal lattices-4096 (SHE)
This work (2048-bit)	**5.31 ms**[(c)]	**about 120 times (31 KByte)**	**ring-LWE-2048 (SHE)**

Only one feature vector is protected (SCiFI [25], Protocol of [2])

Two feature vectors are protected, which condition is tighter (Previous work [31], This work)

† denotes the ratio of (encrypted feature vector size)/(plain feature vector size)
‡ uses a similar packing method as in this work
(a) on an 8 core machine of 2.6 GHz AMD Opteron processors with 1 GByte memory
(b) on an Intel Core 2 Duo 2.13 GHz with 3 GByte memory
(c) on an Intel Xeon X3480 at 3.07 GHz with 16 GByte memory

[2] Osadchy et al, “SCiFI – A system for secure face identification”, IEEE Security & Privacy 2010.
[25] Blanton et al, “Secure and efficient protocols for iris and fingerprint identification”, ESORICS 2011.
[31] Yasuda et al., “Packed homomorphic encryption based on ideal lattices and its application to biometrics”, MoCrySEn 2013.

Conclusions and Future Work

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Conclusions

- We proposed a packing method in the ring-LWE based SHE scheme
  - Main trick: two types of packed ciphertexts
  - It gives us an efficient computation of secure inner product
- Our method gave a practical solution in privacy-preserving biometrics
  - It takes 5.31 ms for secure Hamming distance of 2048-bit vectors
  - Faster than additively schemes even under tighter condition
  - Lattice-based SHE with our packing method would be practical

Future Work

- Application of our packing method in wider applications
  - e.g., secure pattern matching for secret search

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