

DIVIDE-AND-CONQUER SPECTRAL DECORRELATION FOR REMOTE-SENSING IMAGE CODING

Submitted to the Universitat Autònoma de Barcelona in partial fulfillment
of the requirements for the degree of Doctor of Philosophy in Computer Science

by Ian Blanes Garcia

supervised by Dr. Joan Serra Sagristà

Bellaterra, November 2010



I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy.

Bellaterra, November 2010

Dr. Joan Serra Sagristà
(Supervisor)

Committee:

Dr. Josep Rifà i Coma

Dr. Michael Marcellin

Dr. Enrico Magli

Dr. Carole Thiebaut (substitute)

Dr. Philippe Salembier (substitute)

*To my family for their
unconditional support.*

Abstract

In the context of codification of remote sensing images —which are those obtained by air- or space-born sensors— there is a particular kind of images that requires special attention: hyperspectral images, i.e., those that instead of representing a scene in grayscale or using the three primary colors, have hundreds of representations of a spatial location, where each representation corresponds to a small fraction of the light spectrum. Hyperspectral images contain more information than traditional images and allow for, as an example, the remote detection of the ground composition or for the measure of vegetation lushness. Hyperspectral images contain high amounts of redundancy because of the multiple versions of the same location. Taking into account such redundancy may help achieve significant increases in compression factors. A common technique to remove redundancy from hyperspectral images is the use of a spectral transform, like the Karhunen-Loève transform or wavelets. Best results are obtained with the former transform or derived versions; nonetheless, it presents, among other problems, a very high computational cost that prevents its use in environments where computational resources are scant, e.g., a satellite sensor. In order to address this issue, derivations of the Karhunen-Loève transform based on a divide-and-conquer strategy have been introduced. The idea behind these derivations is to decompose the transform into a collection of small parts, and only keep those parts which yield an effective improvement in terms of compression performance. Following such strategy, various decompositions are introduced and exhaustively analyzed in the scope of this dissertation.

En el context de la codificació d'imatges de teledetecció —que són aquelles captades per sensors aerotransportats o de satèl·lit— hi ha una família d'imatges que mereixen especial atenció: les imatges hiperespectrals, o dit d'una altra manera, aquelles que en comptes de representar una imatge en escala de grisos o en els tres colors primaris, el que fan és proporcionen centenars de versions d'una mateixa zona, on cada versió només conté una petita fracció de l'espectre lluminós. Aquestes imatges aporten molta més informació i permeten, per exemple, la detecció de la composició del sòl o la mesura de la frondositat de la vegetació. Tanmateix, el fet de representar múltiples versions d'una mateixa zona afegeix una notable redundància a les dades; redundància que si es tracta adequadament, pot portar a millores significatives dels factors de compressió d'aquestes imatges. Una tècnica habitual per eliminar la redundància de les imatges hiperespectrals és l'ús d'una transformada espectral, com pot ser la de Karhunen-Loève o les wavelet. Els millors resul-

tats s'obtenen amb la transformada de Karhunen-Loève o derivats. Tot i això, aquesta transformada presenta, entre d'altres problemes, un cost computacional molt alt que impedeix que es faci servir habitualment en un entorn amb recursos limitats, com podria ser un sensor de satèl·lit. De cara a afrontar aquest problema, s'introdueixen les derivacions divideix-i-venceràs de la transformada de Karhunen-Loève. Aquestes segueixen el principi de descompondre la transformada en parts menors i només conservar aquelles parts que realment proporcionen un avantatge en termes de compressió. Seguint aquesta estratègia, en aquesta tesi es presenten diverses formes d'aplicar una descomposició i s'analitzen de forma exhaustiva els seus rendiments en l'àmbit de la teledetecció.

En el contexto de la codificación de imágenes de teledetección —que son aquellas captadas por sensores aerotransportados o de satélite— hay una familia de imágenes que merecen especial atención: las imágenes hiperespectrales, o dicho de otra forma, aquellas que en vez de representar una imagen en escala de grises o en los tres colores primarios, lo que hacen es proporcionar centenares de versiones de una misma zona, donde cada versión solo contiene una pequeña fracción del espectro luminoso. Estas imágenes aportan mucha más información y permiten, por ejemplo, la detección de la composición del suelo o la medida de la frondosidad de la vegetación. Así mismo, el hecho de representar múltiples versiones de una misma zona añade una notable redundancia a los datos; redundancia que si es tratada adecuadamente, puede llevar a mejoras significativas de los factores de compresión de estas imágenes. Una técnica habitual para eliminar la redundancia de las imágenes hiperespectrales es el uso de una transformada espectral, como puede ser la de Karhunen-Loève o las wavelet. Los mejores resultados se obtienen con la transformada de Karhunen-Loève o derivados. Sin embargo, esta transformada presenta, entre otros problemas, un coste computacional muy elevado que impide que se use habitualmente en un entorno con recursos limitados, como podría ser un sensor de satélite. Para afrontar este problema, se introducen las derivaciones divide-y-vencerás de la transformada de Karhunen-Loève. Estas siguen el principio de descomponer la transformada en partes menores y solo conservar aquellas partes que realmente proporcionan una ventaja en términos de compresión. Siguiendo esta estrategia, en esta tesis se presentan diversas formas de aplicar una descomposición y se analizan exhaustivamente sus rendimientos en el ámbito de la teledetección.

Preface

This PhD thesis was started in autumn 2008. By then, my supervisor and I had already been able to reproduce experiments from other researchers on the use of the Karhunen-Loève Transform (KLT) for hyperspectral image compression, and had done an extensive review of the state of the art on that topic as part of my master's thesis. At that point, we had several ideas in relation to the KLT that we wanted to try. So we tried them, and, fortunately, one of them worked quite well in practice, i.e., to use a divide and conquer strategy on the KLT when coding hyperspectral images to reduce its computational cost. The strategy of divide and conquer, in its mathematical sense, is not particularly new, as there are known examples of its use dating from at least year 200 B.C. in Babylonia [1, p. 420]; nor is the KLT, with publications describing it by Karhunen in 1946 [2], Loève in 1948 [3], and early work by Pearson in 1901 [4] and Hotelling in 1933 [5]. Yet, it seems that the computational resources needed for the KLT were not available until recently, when it has started to gain widespread adoption in image coding.

Very soon we realized that we were not the only ones working on such strategy for the KLT. Apparently, Y. Wongsawat, S. Orintara, and K. R. Rao were the first ones to work on this topic with a recursive decomposition of the KLT for electroencephalogram data compression dating from 2006, which went under our radar for a while due to the divergent topic. Other authors also started working on a divide-and-conquer strategy, such as Q. Du, W. Zhu, H. Yang, and J. E. Fowler; or J. A. Saghri, S. Schroeder, and A. G. Tescher. It seems that we all were at the right point in time for this development to happen, and all of us worked in parallel with our own perspective on which was the best way to proceed. Each group

of authors provided quite different versions of the same idea, nonetheless, each with its own target scenario and particular drawbacks. This fact was very stimulating for me, as the topic on which I was working, not only was of interest to other researchers worldwide, but I also had a strong motivation to deliver high quality results, knowing that they might be scrutinized by the keen eye of fellow researchers.

On a similar line of thought, we had the impression that it would be more productive and fulfilling if I delivered this PhD thesis in the form of a compendium of publications, as we perceived that it would reach a larger audience, that it would have the extra quality assurance of the peer-review process of each individual publication, and that, on a hot topic such as this one, we would be able to communicate our results earlier and more often. Hence, as the reader will see soon, this PhD thesis is presented as a compendium of publications on this topic, where the authors of the publications are myself and my supervisor.

When reading this compendium, one of the less obvious aspects of our research might be the amount of work that had to be done to produce all the experimental results. Vast amounts of computational resources were used to test our proposed methods and the existing ones on a representative number of scenarios. In particular, I remember a table in one of the publications that summarizes over ten-thousand codings and decodings of hyperspectral images. I was able to perform all the experimental results thanks to the dedicated computational resources provided by our department, and by the Òliba project of our university, which pools together idle resources in student computer labs. Another critical part of the experimental results has been the use of image coding software generously made available by other people, of which I specifically mention Kakadu by D. Taubman, QccPack by J. E. Fowler, and all the software developed by the folks in our research group.

I want also to acknowledge in this preface all the support I have received during the development of this PhD thesis. It has been a pleasure knowing everyone I worked with during that period: all the people from our research Group on Interactive Coding of Images and from our department with whom I shared so many

good experiences, the folks at the Department of Geography and at the Center for Ecological Research and Forestry Applications, the people at the Department of Electronics and Informatics of the Vrije Universiteit Brussel, in particular Peter Schelkens who supervised my stay there, and my PhD supervisor, Joan Serra-Sagrìstà, for his exceptional support and leadership. I must also acknowledge the anonymous peer reviewers of the publications for their valuable comments and suggestions, and the High Technology Imaging Diagnostic Unit of the Parc Taulí Health Corporation, the National Aeronautics and Space Administration, and the U.S. Geological Survey for the images provided for experimentation, with a particular mention to Aaron Kiely for its selfless provision of uncalibrated satellite images.

This work was financially supported in part by the Spanish Government, by the Catalan Government, by the Universitat Autònoma de Barcelona, and by FEDER under Grants TSI2006-14005-C02-01, TIN2009-14426-C02-01, TIN2009-05737-E/TIN, SGR2005-00319, and SGR2009-1224, and under fellowships FI2008, FIUAB2008, and FPU2008.

The cover of this manuscript and the small puzzle pieces scattered inside it are based on the map attributed to Henri Abraham Châtelain, a Paris-born protestant minister, published between 1705 and 1720 entitled “Carte Tres Curieuse De La Mer Du Sud Contenant Des Remarques Nouvelles Et Tres Utiles Non Seulement Sur Les Ports et Isles de Cette Mer.” Map reproduction is courtesy of the Norman B. Leventhal Map Center at the Boston Public Library.



Contents

Abstract	iii
Preface	v
1 Introduction	1
2 Contributions	7
2.1 Regular multilevel	11
2.2 Analysis of alternative quality measures	23
2.3 Dynamic and static multilevel	29
2.4 The Pairwise orthogonal transform	41
2.5 Global comparison	55
3 Conclusions	63
3.1 Summary	63
3.2 Future Research	67
Appendices	68
A List of all produced publications	69
B Acronyms	71
C Implementation particularities	73
Bibliography	77

Chapter 1

Introduction

In the remote-sensing context, images are often acquired with multi- or hyperspectral sensors, which are those sensors that are capable of capturing multiple representations of a spatial region, each one capturing only a finite interval of the light spectrum. Current remote sensors produce large volumes of information, as each sensor gathers many images, and each image may have a size that ranges from tens of megabytes to a gigabyte. Such images, and particularly hyperspectral ones, contain enormous amounts of redundancy between each of their representations of a spatial region, or otherwise known as spectral components.

Mathematical transforms are a relevant topic in the image coding field because of their ability to reorganize information in ways that render the structure of the information more apparent, and thus boost the performance of other image coding stages. In this regard, spectral transforms are mathematical transforms that are designed and used to tackle the removal of spectral redundancy. With the use of image coding techniques, and in particular with the use of spectral transforms, storage and transmission capabilities of remote-sensing systems are improved, thus decreasing costs and improving user interaction with the information.

The most common spectral transforms in hyperspectral image coding are the Karhunen-Loève Transform (KLT) and the family of wavelet transforms. While competitive coding results are achieved with the use of either the KLT or wavelets, the former provides a significantly higher coding performance at the expense of

several significant disadvantages, which in particular prevent the adoption of the KLT in some situations, such as in interactive or real-time processing, and in power- or memory-constrained environments like on-board sensors. In any case, settling the existing issues of the KLT would also improve the scenarios where the transform is already being used.

The identified issues of the KLT in this context are as follows: a high consumption of computational resources both in terms of floating-point operations performed and in terms of peak memory usage; its high implementation difficulty, which is derived from the several numerical issues that have to be taken into account, like algorithm convergence conditions and rounding error accumulation; and the fact that the KLT is not a scalable transform, and thus no partial inversion of the transform is possible for some of the spectral components without having the full coded image. Wavelets, on the other hand, have much lower computational demands and provide component scalability.

The cited disadvantages of the KLT are well known, and several techniques have been proposed in the literature to alleviate them. The Discrete Cosine Transform (DCT) was proposed [6] to overcome the high computational cost of the KLT, using the Fast Fourier Transform and assuming a Toeplitz matrix as data covariance matrix; however, the Discrete Cosine Transform (DCT) yields poor results as a spectral decorrelator [7]. On a related note, the DCT can be extended into the Approximate Karhunen-Loève Transform (AKLT) [8] and the $AKLT_2$ [9] with the use of mathematical first and second order perturbations respectively.

A subsampling method can be used [7] to alleviate transform training costs, which are dominated by the covariance matrix calculation if the spatial size of an image is large enough, and habitually amount to one fifth of the total computational cost. In general, the covariance matrix can be extrapolated from a very small sample of the spatial data of the image, virtually eliminating training costs; however, if the spatial size of an image is significantly small or if multiple transforms are applied in small spatial blocks, then subsampling becomes ineffective and eigendecomposition costs are more significant. In this situation, methods such as [10] might be of use to reduce the eigendecomposition cost. A third method to di-

minish training costs, is to pretrain a transform—or also to create a codebook of transforms—for a generic corpus of images [11, 12]. Pretraining eliminates training costs in all scenarios, but it has the drawback that it produces inferior coding gains because of the lack of specificity.

Three additional techniques in relation to the KLT are also worth noting. The first one, in a lossless coding scenario, is to substitute multiplication operations in the forward and inverse transforms by incorporating them into the lifting decomposition in form of addition and shift operations [13, 14]. The second technique, is to reject the Gaussian source assumption, and improve coding performance by trying to find the optimal spectral transform with an algorithm based on Independent Component Analysis [15, 16]. Pretraining can be used to avoid the cost of the Independent Component Analysis [17, 18]. Finally, the third technique is the Distributed KLT [19], which assumes a model of multiple small local forward transforms and a global inverse transform, and finds a local optimum to the overall setup using an iterative process.

A recent trend to address the issues of the KLT is to use divide-and-conquer techniques. The idea behind these techniques is that as the dominant term of the KLT cost is quadratic, multiple smaller transforms have substantially less cost than a larger one. In a full KLT, all components are perfectly decorrelated with each other, while in a divide-and-conquer strategy, components are decorrelated only if a high amount of energy is shared between them, and ignored otherwise, as parts with low energies are expected to have lower influence on coding performance. If multiple smaller transforms are arranged in a way that significant outputs of some of them are further processed by others, then global decorrelation may still be achieved through a combination of small-sized transforms.

The publications in this compendium belong to this last trend. We present new multilevel divide-and-conquer strategies that address the computational cost issue of the KLT or that are able to trade a moderate coding performance penalty for the ability to operate on environments with strong constraints on computational resources. In the literature, there are three other contributions by other authors to the divide-and-conquer trend. First, the recursive strategy is presented in [20, 21].

Second, the plain clustering strategy is discussed in [22]. And third, the two-level strategies are introduced in [23, 24]. A review and side-by-side comparison of these three strategies and the ones presented in this compendium can be found in [25].

The contributions of this compendium are four publications in some of the most relevant journals and conferences on this topic. Chronologically, they are:

- I. Blanes and J. Serra-Sagristà, "Clustered reversible-KLT for progressive lossy-to-lossless 3d image coding," in *IEEE Data Compression Conf. 2009 (DCC 2009)*. IEEE Press, pp. 233–242, Mar. 2009.
- I. Blanes and J. Serra-Sagristà, "Quality evaluation of progressive lossy-to-lossless remote-sensing image coding," *ICIP 2009. Proceedings of 2009 IEEE International Conference on Image Processing*, pp. 3709–3712, Nov. 2009.
- I. Blanes and J. Serra-Sagristà, "Cost and scalability improvements to the Karhunen-Loève transform for remote-sensing image coding," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 48, no. 7, pp. 2854–2863, Jul. 2010.
- I. Blanes and J. Serra-Sagristà, "Pairwise orthogonal transform for spectral image coding," *IEEE Transactions on Geoscience and Remote Sensing*, 2010, in press.

Each of the publications provides an incremental improvement on the field of divide-and-conquer spectral decorrelation. The first publication is where the multilevel strategies are initially proposed to reduce the KLT computational cost. In the second publication, previously unexplored gaps, in the context of hyperspectral image coding, in relation to the evaluation of spectral transforms are addressed. The second publication provides guidelines on how, in order to be useful to the remote-sensing community, results regarding the research on remote-sensing image coding shall be reported. Later, in the third publication, multilevel strategies are severely refined with the introduction of eigentersholding methods, which drive down further computational costs and achieve satisfactory component scalabilities. In the fourth publication, a new spectral transform is proposed —the

Pairwise Orthogonal Transform (POT)— that, using the principles of a multilevel strategy, settles all the cited issues of the KLT. The POT is able to overcome the issues by using a minimalistic structure that yields lower coding performance than the KLT, yet higher than wavelets.

A brief discussion on the quality and relevance of the publications is now provided. The two first publications appear in the proceedings of the two most relevant conferences in the image compression field, i.e., the Data Compression Conference (DCC), and the International Conference on Image Processing (ICIP). The IEEE DCC is perhaps the most prestigious conference for the academy for data compression topics¹. The IEEE ICIP is relevant for its large number of attendees and a very strong industry participation². Regarding the last two publications, which are journal papers published in the IEEE Transactions on Geoscience and Remote Sensing, note that while standard ISI impact factors are not yet available for the year of publication, data from the previous four years ranks this journal in the first quartile of both the “Engineering, Electrical & Electronic” and “Remote Sensing categories”, with positions 34 of 246 and 4 of 21 respectively during the year 2009³.

¹Appears on ISI Proceedings indexed as Proceedings; has a factor of 0.97 over 1 in the Computer Science Conference Ranking <http://web.archive.org/web/2008/www.cs-conference-ranking.org/conferencerrankings/alltopics.html>; and has an A⁺ in the 2008 Ranking of the Computing Research and Education Association of Australasia <http://core.edu.au/cms/images/downloads/conference/Astar.pdf>.

²The 2009 ICIP edition had over one thousand reviewers from over 550 different institutions, a large portion of them were part of the industry. http://icip2009.org/ICIP2009_ProgramGuide.pdf

³Journal Citation Reports, <http://www.isiknowledge.com>

Chapter 2



Contributions

In this chapter, the research results of this compendium are presented and discussed. The principal result are four multilevel strategies to perform divide-and-conquer decorrelation in hyperspectral images, where each strategy provides a different set of features, trade-offs, and target scenarios. Other results include an analysis of suitable evaluation techniques for these structures, and a set of open-source software implementations of the methods and techniques used to produce the results presented in this compendium.

Hereafter, the four multilevel strategies are described.

- The **regular multilevel** strategy is the first of the multilevel strategies and the simplest one. It was proposed in [26], showing that multilevel strategies were able to substantially reduce the cost of the KLT while obtaining similar coding performance results. Special attention was paid to its use with the Reversible Karhunen-Loève Transform (RKLT) [27, 28] for lossless or progressive-lossy-to-lossless image coding within the JPEG2000 coding system [29, 30].

The idea behind the regular multilevel strategy is to divide a full transform into small and much more affordable clusters, and then gather the most significant half of each cluster into a new level, where the same procedure is applied recursively. An example of such structure is provided in Fig. 2.1a, where 16 components are decorrelated with this strategy.

- The **dynamic multilevel** strategy, introduced in [31], combines the ideas of regular multilevel with eigenthresholding algorithms in order to adjust the components that are forwarded to the next level at runtime, while the transform is applied. Eigenthresholding algorithms fit precisely in this situation, as they are methods that try to determine the amount of significant components after a KLT. An example of a dynamic multilevel structure is shown in Fig. 2.1b.
- Also introduced in [31], the **static multilevel** strategy follows the principles of regular structures but with additional degrees of freedom, i.e., the size of clusters and the amount of components forwarded to next stages may vary among levels. With a combination of eigenthresholding methods and exhaustive search, the additional degrees of freedom can be tackled to produce structures very well suited to process images from a specific sensor or training corpus. In Fig. 2.1c, a static multilevel structure is shown.
- Finally, the fourth multilevel strategy is the **Pairwise Orthogonal Transform (POT)** [32]. The POT is a minimalistic multilevel structure that, due to its minimalistic condition, gains a set of derivate properties that make it suitable for resource constrained environments while still providing a medium coding performance. One of such properties is that as it requires a very small amount of side information because of its small structure, the transform can be applied in small blocks or line by line, and hence with a very small memory requirement. The POT is a structure composed only by clusters of two components, where of the two outputs of each cluster, one is forwarded to the next level while no further decorrelation is applied to the other one. In Fig. 2.1d an example of the POT is shown, and in Fig. 2.2, for clarity, an example of a line-based application of the POT is provided.

The purpose of remote-sensing images is often to be part of a scientific experiment where these images are handled by an automated process, for example, when a spectral unmixing algorithm is used to determine the location and abundance of

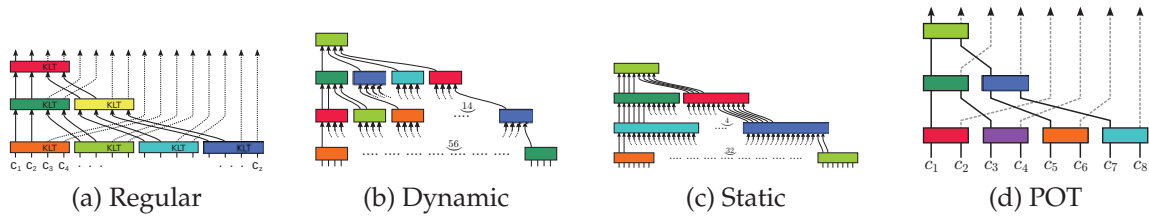


Figure 2.1: Examples of multilevel strategy for spectral decorrelation

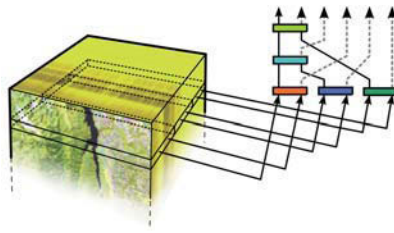


Figure 2.2: An example of a line-based application of the POT.

materials in a hyperspectral image. For this reason, it is of interest to know the effects of lossy image compression to a possible posterior automated stage. Some alternative quality measures have appeared in the recent literature [33, 34, 35]. Bearing this in mind, an additional contribution of this compendium is an **analysis of alternative quality measures** with the objective of evaluating their suitability in assessing the quality of proposed hyperspectral coding techniques [36]. Of particular interest is the conclusion that some of the evaluated alternative quality measures in the tested scenario are highly correlated with the traditional ones based on Mean Squared Error (MSE). The alternative measures that have been found to be most relevant are the ones based on the variation of results of an unsupervised classifier. In this case, the unsupervised classifiers have been: a k-Means classifier, which is a very common clustering approach [37] based on an iterative partitioning process around cluster centroids; and an RX anomaly detector [38], which is also a very common outlier-detection procedure in remote sensing that works by measuring, through the Mahalanobis distance, the differences between spatial locations and background. Following the guidelines set by [36], our publications posterior to that study include comparisons using classification-based measures.

During the development of the thesis, three **software tools** were developed and released as open-source software.

- The **spectral transform software** [39] is an implementation of the Karhunen-Loève Transform to be used as spectral decorrelator, with many additional features, such as, matrix factorizations for reversible integer mapping [27, 28], covariance subsampling [40, 7], or clustered and multi-level transforms [26, 31].
- The **alternative metrics package** [41] is an implementation of various alternative metrics for hyperspectral images. Metrics are grouped in two families of metrics: statistical and classification-based. The family of statistical metrics is formed basically by extensions of alternative bi-dimensional metrics from [33]. The following statistical metrics are implemented: Minimum Spectral Pearson's Correlation [33], Maximum Spectral Similarity (MSS) [34], Maximum Spectral Angle (MSA) [33], Spectral Wang-Bovik Q [35, 33], and Spectral Fidelities [42, 33] The following two classification-based measures are also implemented: k-Means classification [37], and Reed-Xiaoli (RX) anomaly detection [38].
- An **efficient implementation of the Pairwise Orthogonal Transform** [43], using a single-thread implementation of a consumer/producer model based on the observer pattern, where the transform is applied line-by-line and minimal memory is required.

The rest of this chapter is organized as follows. The next four sections reproduce the articles of this compendium, and a fifth section includes a global comparison of all the multilevel structures.

2.1 Regular multilevel

```
@inproceedings {BS09,  
  AUTHOR = {I. Blanes and J. Serra-Sagristà},  
  TITLE = {Clustered Reversible-KLT for Progressive Lossy-to-Lossless  
    3d Image Coding},  
  BOOKTITLE = {IEEE Data Compression Conf. 2009 (DCC 2009)},  
  PUBLISHER = {IEEE Press},  
  MONTH = {Mar.},  
  YEAR = {2009},  
  PAGES = {233-242},  
  ISSN = {1068-0314},  
  DOI = {10.1109/DCC.2009.7},  
}
```


2.2 Analysis of alternative quality measures

```
@inproceedings {BS09b,  
  AUTHOR = {I. Blanes and J. Serra-Sagristà},  
  TITLE = {Quality evaluation of progressive lossy-to-lossless  
    remote-sensing image coding},  
  BOOKTITLE = {2009 IEEE International Conference on Image Processing (ICIP 2009)},  
  PUBLISHER = {IEEE Press},  
  MONTH = {Nov.},  
  YEAR = {2009},  
  PAGES = {3709-3712},  
  ISSN = {1522-4880},  
  DOI = {10.1109/ICIP.2009.5414283},  
}
```


2.3 Dynamic and static multilevel

```
@article {BS10a,  
  AUTHOR = {I. Blanes and J. Serra-Sagristà},  
  JOURNAL = {IEEE Transactions on Geoscience and Remote Sensing},  
  TITLE = {Cost and Scalability Improvements to the Karhunen-Loève Transform  
    for Remote-Sensing Image Coding},  
  VOLUME = {48},  
  NUMBER = {7},  
  PAGES = {2854-2863},  
  MONTH = {Jul.},  
  YEAR = {2010},  
  ISSN = {0196-2892},  
  DOI = {10.1109/TGRS.2010.2042063},  
}
```


2.4 The Pairwise orthogonal transform

```
@article {BS10b,  
  AUTHOR = {I. Blanes and J. Serra-Sagristà},  
  JOURNAL = {IEEE Transactions on Geoscience and Remote Sensing},  
  TITLE = {Pairwise Orthogonal Transform for Spectral Image Coding},  
  VOLUME = {PP},  
  MONTH = {Oct.},  
  YEAR = {2010},  
  ISSN = {0196-2892},  
  DOI = {10.1109/TGRS.2010.2071880},  
}
```


2.5 Global comparison

The objective of this section is not to justify the results of the proposed methods, which has already been done in their respective publications, but to present a global comparison of all of them in a common framework so that the reader can observe, if she or he so wishes, the relation among the different proposed methods. In addition, a brief qualitative summary is included showing the key aspects of the proposed transforms in relation to all the other divide-and-conquer transforms present in the literature. To that purpose, the presented decorrelation methods are all combined with JPEG2000 [29], in this case as implemented by Kakadu Software [44], and results are provided for the hyperspectral image AVIRIS Yellowstone scene 0 [45] (other images yield qualitatively similar results). The AVIRIS Yellowstone scene 0 image is captured by a 16 bpppb air-borne sensor, has 224 spectral components, and a spatial size of 677 columns by 512 rows.

The following three facets of each methods are analyzed: **coding performance**, in this case defined either by the Signal-to-Noise Ratio (SNR) where σ^2 is the variance of the original image and $\text{SNR} = 10 \log_{10}(\sigma^2/\text{MSE})$, or by the Preservation of Classification (POC) of the k-means and RX classifiers, which corresponds to the percentage of spatial locations that stay in the same class after coding distortion is introduced; **computational cost**, in this case defined by the amount of floating point operations required to train, and apply the forward and inverse transforms; and **component scalability**, in this case defined by the quantity of components in the transformed domain that are required to recover one component from the original domain. First, these three facets are analyzed independently, and then the trade-offs between them for each transform are considered.

Coding performance results are presented for both lossy and progressive lossy-to-lossless in Fig. 2.4 and Fig. 2.5 respectively. Classification-based results are clearly more unstable than SNR-based ones. This is both due to the nature of classifiers themselves and to the fact that the coder employs a Rate-Distortion Optimization method which uses the MSE as distortion. Nonetheless, classification- and SNR-based results show similar trends with the KLT providing best results,

closely followed by ML regular and ML static, and at a larger distance by ML dynamic, POT and the wavelet. Results are presented as the performance difference between each particular method and the KLT, and because the KLT requires more side information than all the other methods, at very low bitrates, where the KLT codification is still coding side information, other methods start sooner to produce image values, and thus produce a slight improvement in its coding performance. This improvement is more relevant for transforms with less side information, and as lossless KLT derivatives require a larger portion of side information, the difference is more emphasized in that case. Note that the performance of classifiers seems to saturate around 2.5 bpppb, with not much improvement after that position as bitrate increases. Another interesting issue appears on progressive-lossy-to-lossless codings, where because the larger lifting decompositions of more complex transforms introduce greater quantization errors, simpler transforms experience a relative performance boost at medium to high bitrates.

When analyzing the computational-cost facet of the proposed transforms (see Fig. 2.3), a notable computational cost reduction is produced in all of them. The transform training cost is virtually eliminated by the use of the covariance subsampling optimization proposed in [40], except for the POT, for which such optimization is not applicable because of its line-by-line operation. In the lossless case, transform costs are approximately fifty percent higher than for their lossy counterparts due to the use of quasi-complete pivoting in the lifting decomposition as introduced in [28], which improves the overall coding performance, in particular for larger transforms. In relation to wavelets, ML dynamic, and POT demand very similar costs.

As for component scalability of the transforms, a comparison is reported in Table 2.1. The KLT does not provide any component scalability, thus, as reported in this case, to recover one spectral component, all of the spectral components must be available in the transformed domain. On the other hand, multilevel transforms improve their component scalability as they become simpler. Wavelet component scalability is relatively poor, in particular for the CDF 9/7.

Once each of the three individual facets of each transform has been analyzed, a

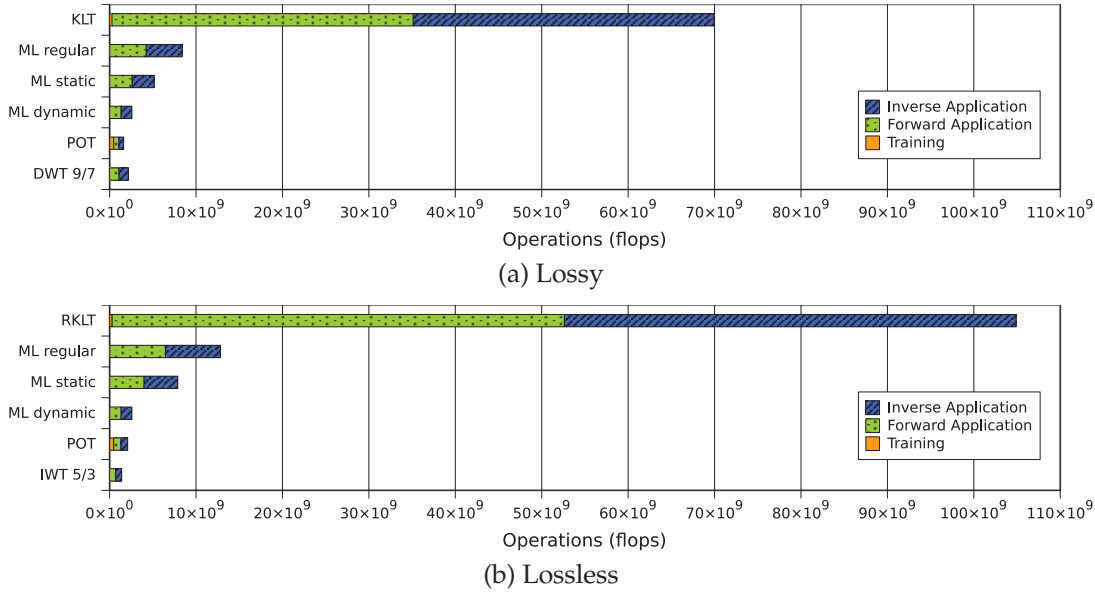


Figure 2.3: Transforms computational cost.

global comparison of the trade-offs between them is introduced in Fig. 2.6, which might be understood in a way similar to a traditional Rate-Distortion where in this case instead of Rate and Distortion, three other features are compared. One of the perhaps more interesting observations is that multilevel transforms form a relatively straight line in the trade-off space both for lossy and for lossless. In comparison, the CDF 9/7 wavelet is below the other transforms in terms of either coding performance vs. speed, coding performance vs. scalability, or speed vs. scalability. On the other hand, if scalability is not taken into account, the CDF 5/3 provides a trade-off relation competitive with that of the other transforms, being the transform with lowest cost and lowest performance. A summary of the key aspects of the proposed methods as discussed in this section is included in Table 2.6, where, in addition, all the other divide-and-conquer strategies present in the literature are also included.

Table 2.1: Transform scalability (in components required to recover one component). Reported wavelet scalability may be reduced on transform edges due to coefficient mirroring (up to a half).

	Avg.	Min.	Max.
KLT	224.0	224	224
ML regular	42.0	42	42
ML static	38.0	38	38
ML dynamic	13.1	12	15
POT	8.9	8	9
Wavelet CDF 9/7	36.0	32	38
Wavelet CDF 5/3	16.0	11	17

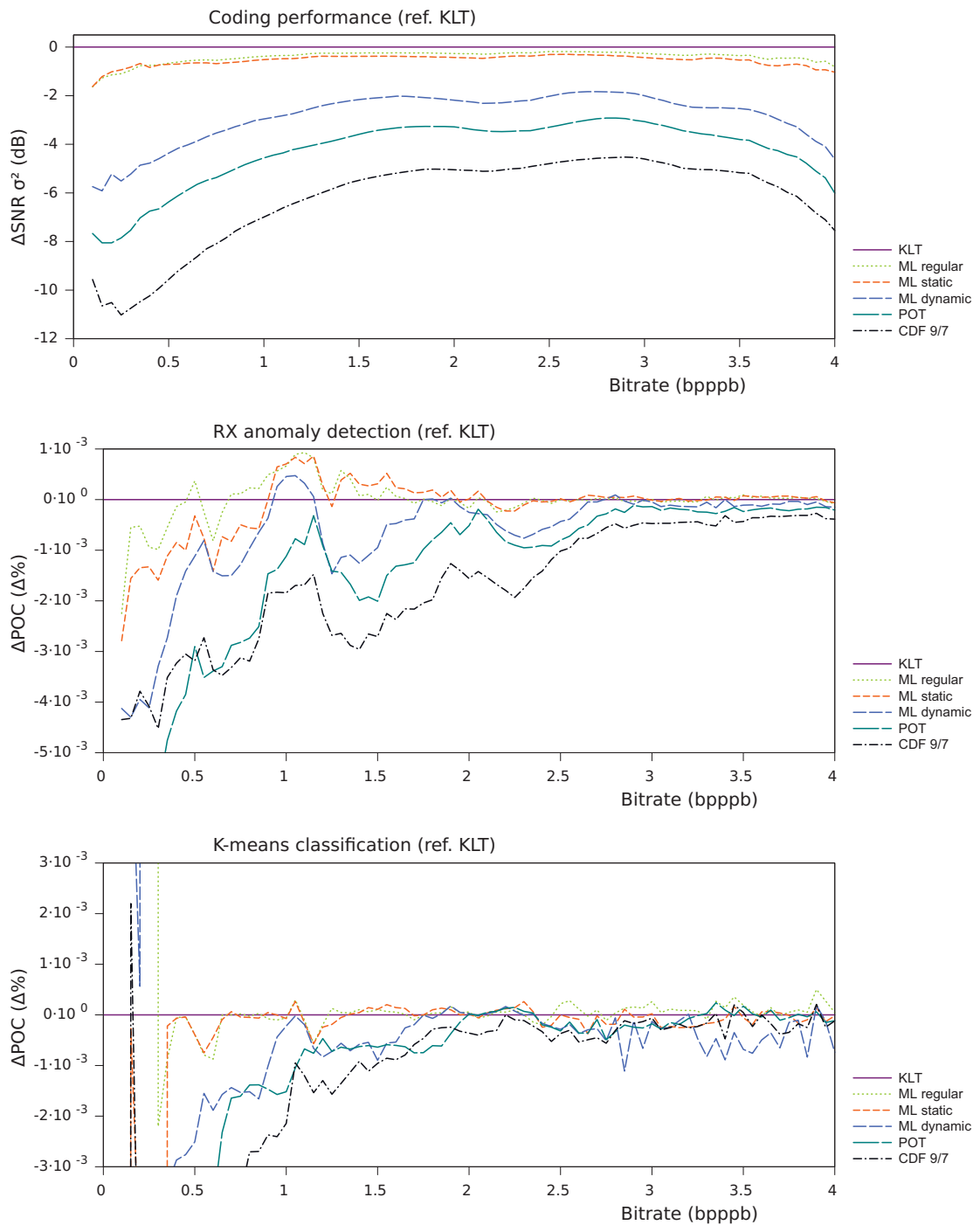


Figure 2.4: Lossy coding performance of the multilevel structures and a wavelet transform in relation to the KLT.

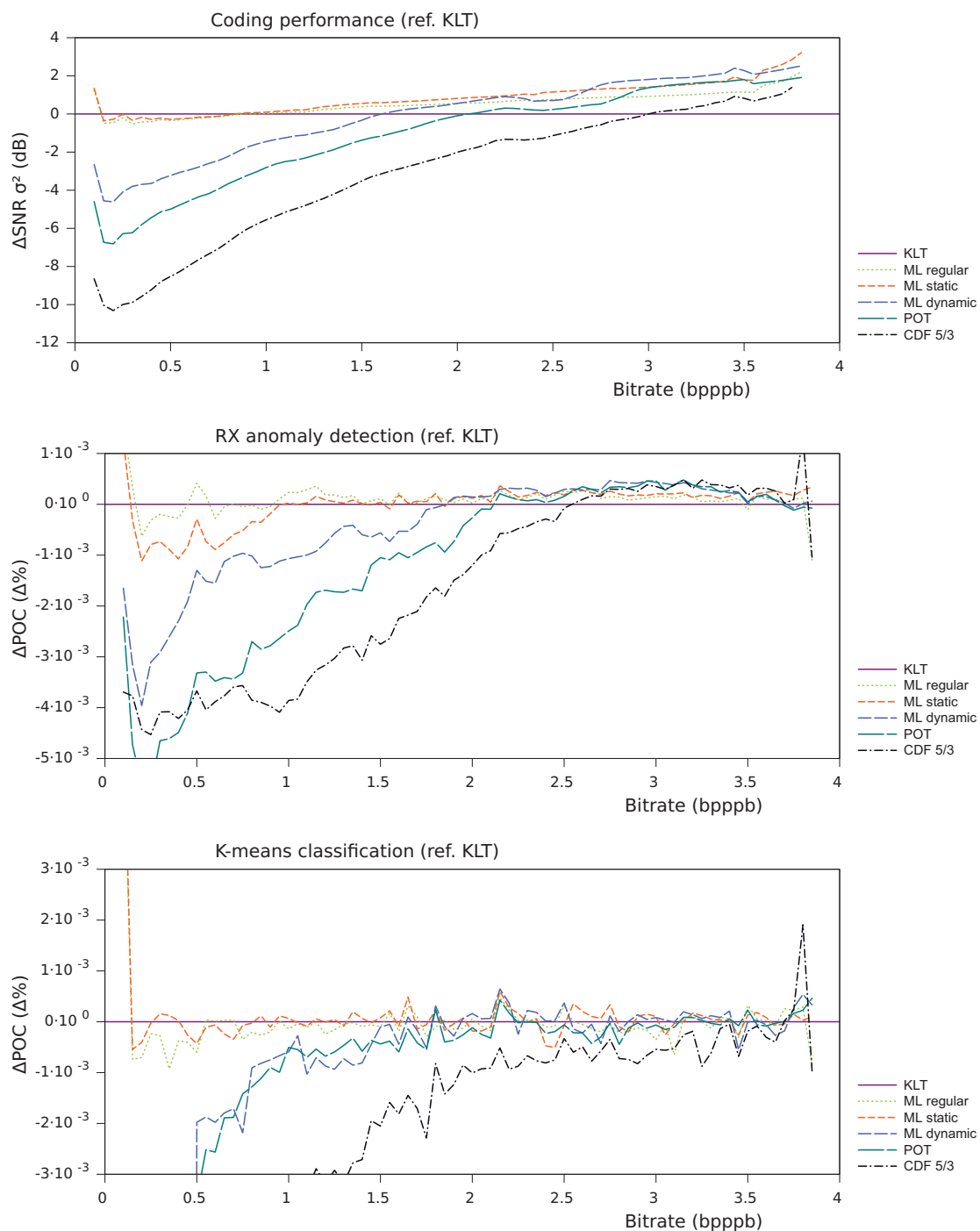
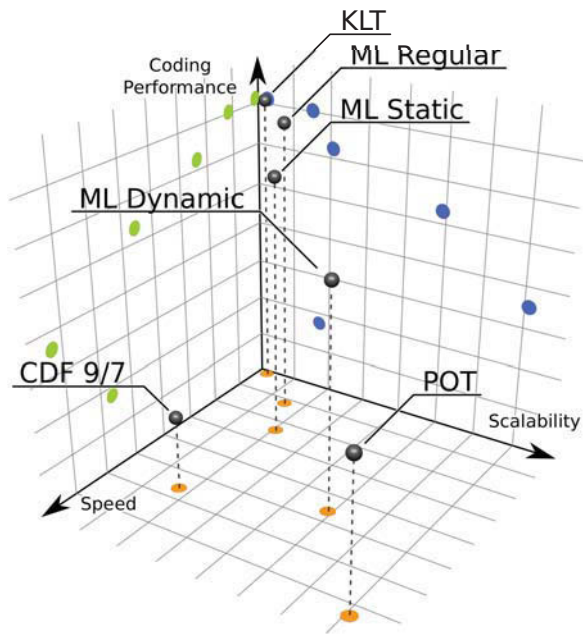
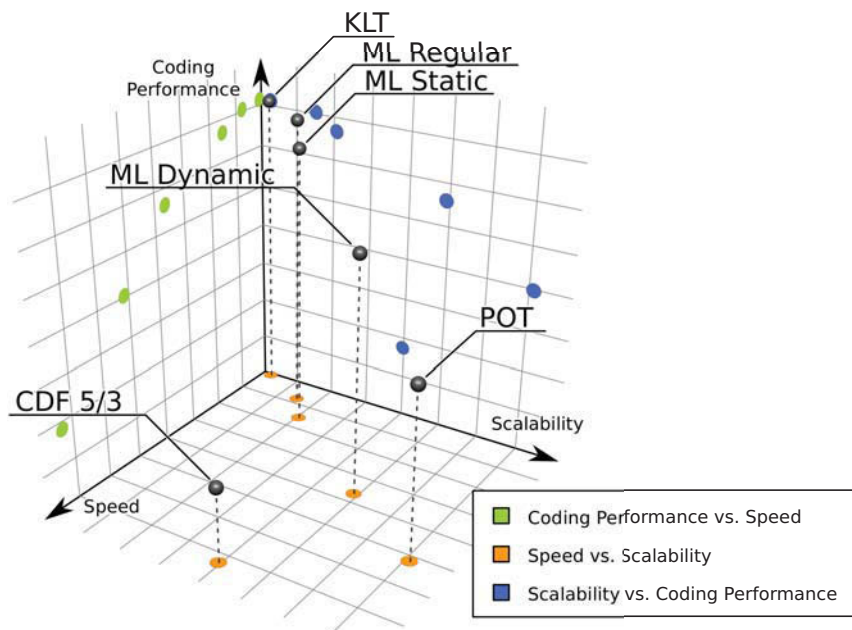


Figure 2.5: Progressive-lossy-to-lossless coding performance of the multilevel structures and a wavelet transform in relation to the KLT.



(a) Lossy



(b) Lossless

Figure 2.6: Comparison of the transforms trade-offs among: coding performance, computational cost, and component scalability.

Table 2.2: Qualitative summary of all the divide-and-conquer strategies. Transforms are ranked from --- (very low) to +++ (very high), according to quantitative data from [25].

	Authors	Publication Date	Reference(s)	Coding Performance (higher is better)	Computational Cost (lower is better)	Component Scalability (higher is better)	Memory Requisites (lower is better)
KLT ¹	Karhunen, Loève	1946	[2, 3]	+++	+++	---	+++
Recursive ²	Wongsawat, Orintara, Rao	2006	[20, 21]	+++	++	---	+++
Static Two-level ³	Saghri, Schroeder, Tescher	2009	[23, 24]	++	-	---	+++
Dynamic Two-level ³	Saghri, Schroeder, Tescher	2009	[23, 24]	++	--	+++	++
Regular Multilevel ⁴	Blanes, Serra-Sagrìstà	2009	[26]	+++	-	+	++
Static Multilevel ⁴	Blanes, Serra-Sagrìstà	2010	[31]	+++	-	+	++
Dynamic Multilevel ⁴	Blanes, Serra-Sagrìstà	2010	[31]	++	--	++	++
Pairwise Orthogonal Transform ⁴	Blanes, Serra-Sagrìstà	2010	[32]	+	---	+++	--
Single-level ⁵	Du, Zhu, Yang, Fowler; Blanes, Serra-Sagrìstà	2009	[22, 26]	++	+	++	++
Wavelet CDF 9/7 ⁶	Cohen, Daubechies, Feauveau	1992	[46]	-	---	+	---
Wavelet CDF 5/3 ⁶	Cohen, Daubechies, Feauveau	1992	[46]	-	---	++	---

Key aspects:

¹ Provides optimal decorrelation, but very high computational cost and no component scalability.

² The use of a recursive strategy, is the first divide-and-conquer strategy for the KLT. Uses two half-size KLT transforms instead of a full one, and the outputs of both half-size transforms are processed again by a third half-size KLT. The process is applied recursively to each of the three half-size KLTs.

³ These structures have two levels; on the first one, multiple transforms provide local decorrelation, while on the second level global decorrelation is provided by processing the most significant outputs from the transforms of the first level. A key aspect is that components on the second level are processed in clusters according to the significance they had on the previous level (the most significant outputs from each transform on the first level are processed together on the second level, the second most significant outputs are processed together, and so on and so forth). Static Two-level has a fixed structure determined by empirical evidence, while Dynamic Two-level has a semi-fixed structure, where inputs of second-level clusters are pruned if they have small influence on the first output of the second-level cluster.

⁴ Smaller transforms are organized in multiple levels, where at each level components are locally decorrelated by smaller transforms, and only a portion of the outputs of each transform is forwarded to a next level. Regular Multilevel has smaller transforms that are equal in size and forward half of their outputs to a next level. Static Multilevel is similar to Regular Multilevel, but transform sizes and forwarded components may vary among levels. Dynamic Multilevel has a fixed transform size, but number of forwarded components varies according to an eigenthresholding function. Pairwise Orthogonal Transform is a minimalistic transform based on only two-component transforms that can be applied line by line due to its very low amount of side information. Note that although Static Multilevel reduces the computational cost of Regular Multilevel and improves its component scalability, they both belong to the same overall cost and scalability categories.

⁵ A plain clustering strategy, where a single level of local transforms is applied. At very low bitrates, it improves coding performance by having a reduced amount of side information. At higher bitrates, it has a poor trade-off between coding performance and computational cost.

⁶ Wavelets are a well known family of transforms, commonly used as an alternative to the KLT as spectral decorrelator. They have a very low computational cost, and do not require a training stage. The CDF 9/7 is the preferred wavelet for lossy image coding, while the CDF 5/3 is preferred for lossless.

Chapter 3

Conclusions

3.1 Summary

In this compendium, several contributions are presented that advance the field of hyperspectral image coding. The contributions are focused on reducing or mitigating several issues that appear when the KLT is used as spectral decorrelator, and that hinder a widespread adoption of that transform in different scenarios, or otherwise reduce the value of the overall transform.

The contributions pertain to the recent trend of using a divide-and-conquer strategy on the KLT procedure to split it into multiple small stages that are applied only where they have a significant effect. In this regard, four different strategies are presented, i.e., regular multilevel, static multilevel, dynamic multilevel, and the Pairwise Orthogonal Transform (POT). Each strategy has its own particularities. The regular multilevel strategy, which was the first of the multilevel strategies, provides a very comprehensive decorrelation while reducing the computational cost of the KLT. Without much *a priori* knowledge of the images to be decorrelated, the use of this transform is a good choice as it provides features very similar to the KLT, but at a reduced computational cost. If further information about the image properties is known, such as the sensor employed to acquire it, then a static multilevel strategy can be of use. A static multilevel strategy is built for a specific training corpus which guides the transform into decorrelating parts

where their application will have more effect, and ignore other parts where little gains can be obtained. With this training, a much more efficient strategy than a regular multilevel strategy is achieved, still matching the coding performance of the KLT. The use of a static multilevel strategy is advisable for scenarios of satellite image archival or in general in scenarios where the image properties are known. On the other hand, if some image properties cannot be assumed but important cost reductions are still required, then the use of a dynamic multilevel strategy is appropriate. The dynamic multilevel strategy is similar to the static multilevel strategy in that it adapts itself to the particular characteristics of an image, although it does so without a specific training corpus and on runtime. Due to the on-runtime nature of the adaptation, a minor performance penalty is produced. Finally, a fourth strategy is introduced, the POT, which targets scenarios where computational resources are scant both in terms of computing speed and memory availability. The POT is a minimalistic transform that has very few side information, and because of this, it can be applied in a line-based mode, with very low memory needed. Target scenarios might be on-board sensors or embedded hardware. An in-depth comparison of the divide-and-conquer techniques was included in Section 2.5, in particular in Table 2.2.

In addition, in order to deliver the main contributions of the compendium, which are the multilevel strategies, other indirect contributions were required, i.e., an analysis of the proper evaluation tools for this scenario, and several open-source software to produce all the experimental results. The conclusion from the analysis of evaluation tools is that for this particular scenario, of all the evaluation methods available, the more relevant ones are those based on the Mean Squared Error (MSE), and those based on classification experiments. Following that conclusion, our subsequent journal papers report results with Signal-to-Noise Ratio (SNR) variance, which is an MSE-based measure, and with classification experiments using a k-means classifier and an RX anomaly detector. Among the software tools developed are the spectral transform package [39], the alternative metrics package [41], and a very efficient POT implementation [43]. The first software tool is an implementation of all the techniques relevant to this research, which include:

lossy Karhunen-Loève Transform (KLT), matrix factorizations for reversible integer mapping [27, 28], covariance subsampling [40, 7], plain clustering [26, 22], the recursive KLT [21, 20], two-level strategies [23, 24], multilevel transforms [26, 31], the POT [32], and several other features such as dependency tracking on component scalability. The second software tool is a measuring tool including the following measures: minimum spectral Pearson's correlation [33], Maximum Spectral Similarity (MSS) [34], Maximum Spectral Angle (MSA) [33], spectral Wang-Bovik Q [35, 33], spectral fidelities [42, 33], k-Means classification [37], and Reed Xiaoli (RX) anomaly detection [38]. The third tool is a proof of concept of a very efficient implementation of the POT designed to show that the theoretical properties can also be reached in practice. Other more tangential publications were also produced, and have been listed in Appendix A.

From the global perspective, when all the contributions on the divide-and-conquer line of research are taken into account [25], one can see that there are some issues that are resolved alike in the different strategies, other issues that are resolved in distinct ways, and one issue that remain unsettled.

The most relevant of the shared features among all strategies is that all of them provide similar ways to achieve lower computational cost and some degree of component scalability by using a collection of smaller KLT transforms, even if some strategies with more success than others. In this regard, all transform share the common idea that the overall computational cost of decorrelation can be reduced by using a collection of smaller versions of a global transform, due to the quadratical reduction in the computational cost as the size of the global transform decreases. Strategies are also similar in that all propose an organization of smaller transforms, where all of the smaller blocks are trained locally as isolated KLTs.

Another interesting issue is how adaptability is achieved. Three different arrangements are observed. For multilevel or two-level static strategies, transforms are adapted to a particular kind of images by empirical experimentation on a training corpus, i.e., a large portion of the possible variations of a static strategy are all tested on a training corpus and the best ones are selected manually. Static strategies are efficient due to its large specialization, which in turn might also be a drawback

on its own. Both dynamic multilevel and dynamic two-level provide on-runtime adaptation (i.e., a transform is modified while each image is being coded), but with two completely different approaches. In the former, the outputs of local transforms are analyzed for relevance using eigenthresholding methods and irrelevant outputs are discarded, while, in the latter, relevance is analyzed at transform inputs by its weighted contribution to the main transform outputs.

The use of interleaving permutations is also a distinctive feature between strategies. Its use enables more efficient transform blocks by grouping components that are not adjacent but are expected to be similar, and thus improve the global transform performance. On multilevel strategies, permutations are not used, and because of that, local decorrelations in advanced stages usually include some components with no correlation between them (but with correlation to other components being decorrelated). Counter-intuitively, for multilevel strategies this is not a significant overhead, and by not using permutations, better component scalabilities are achieved as the components are decorrelated in more localized regions.

The POT provides two additional improvements over the other structures, line-based application and an algebraic eigendecomposition. Both improvements are due to its structure with only two-component transforms. The former improvement requires a structure with very few side information, so that the side information for each line does not penalize more than the improvement produced by the use of the transform. The latter improvement—an algebraic eigendecomposition—simplifies the eigendecomposition process and gives guarantees on the correctness and precision of the outcome.

A issue that remains unsettled is the one of global training. With the use of covariance subsampling, it seems that it might be possible to obtain a global model of the image, to train a global KLT, and to optimize the decomposition of the full transform into smaller ones with extra knowledge provided by their global model that would improve the decomposition over the ones where transforms are trained locally. Such global decomposition would face its own challenges, because, for a given transform, the coding performance estimated theoretically and the real one might be too apart to drive a successful optimization process past an average per-

formance. For example, using a Toeplitz matrix as a theoretical covariance model suggests that a DCT is as good as a KLT, but in practice such theoretical model does not always hold [7]. On the other hand, a more comprehensive theoretical model might be less suitable for optimization.

3.2 Future Research

Following with the previous discussion on the improvements provided by POT over other multilevel structures, perhaps a worthwhile line of research might be to extend the idea of a line-based application to transforms other than the POT, even if other transforms are less likely to benefit from it as much as the POT. A first step would require to obtain a significant reduction on the side information of the transform, for example, by the use of a codebook of the most common training solutions of the KLT, which would be used to represent an approximation of the trained transform by the index of the most similar codebook entry. Similarly, another improvement of the POT, i.e., its algebraic eigendecomposition could perhaps also be extended to other transforms. Apart from the intrinsic benefits of an algebraic eigendecomposition, its use could be combined, as is also done in the POT, with a quantization of the independent variables that represent the transform. With this combination, a side information reduction would occur, and a codebook of training solutions might not be needed. The extension of this second improvement of the POT to other transforms is limited by the availability of algebraic solutions for polynomial root finding, which are well-known for polynomials of degree up to four, and proven to not generally exist for higher degrees, which might limit the improvement to be extended only to transforms that operate in blocks of up to four components. In this regard, the dynamic multilevel strategy, which has a block size of four, seems the one best suited to be extended with these two improvements, or perhaps a new hybrid transform between the POT and the dynamic multilevel strategy with a block size of three might yield good results.

There are several other lines of investigation and applications of this research, with ones more immediate than others. A natural research line is to study the ap-

plication of the divide-and-conquer strategies to 2D image coding, with the POT being the natural candidate, since it has the multilevel structure of a Haar wavelet transform, and since it requires low side information, which is an important requirement for 2D image coding. In this case, due to the lower dimensionality of bi-dimensional images, a line-based approach would not work, as there would be only one training sample per transform, and thus different organizations of the training regions would have to be analyzed. The use of fixed size square blocks, such as the ones commonly used with the DCT, might be an option, or perhaps the transform can smoothly change from one training solution to another at different locations by interpolating its side information between a set of reference keys, which would be points on the original image for which side information would be available.

It is also intriguing whether divide-and-conquer strategies can also be extended to other fields not related to image coding where the KLT is used. For example, one of the proposed transforms might be used as a fast dimensionality reduction before a classification stage (e.g., a neuronal network, or k-NN). In a similar fashion, the more simpler transforms, such as the POT, perhaps could also be used in other fields where wavelets are currently being used, for example, in texture analysis, or in noise reduction.

Another, less immediate, line of research is the decomposition of a global transform into smaller transforms as described above, but without local training on each smaller part. Instead, training would be global and based on the information provided by a global estimation of the signal properties by means of a subsampling optimization. This last line of research is of particular interest because, if successful, it could allow for a generic approximate matrix multiplication operation, where a significant portion of the operations could be omitted depending on the particular characteristics of the matrix being multiplied.



Appendix A

List of all produced publications

(in chronological order)

1. I. Blanes, A. Zabala, G. Moré, X. Pons, and J. Serra-Sagrìstà, "Classification of hyperspectral images compressed through 3D-JPEG2000," in *Springer Lecture Notes in Artificial Intelligence, Lecture Notes in Computer Science*, I. Lovrek, R. J. Howlett, and L. C. Jain, Eds., Knowledge-Based Intelligent Information and Engineering Systems: KES 2008. Zagreb, Croatia: Springer-Verlag Berlin Heidelberg, Sept. 2008, pp. 416–423.
2. I. Blanes and J. Serra-Sagrìstà, "Clustered reversible-KLT for progressive lossy-to-lossless 3d image coding," in *Data Compression Conf. 2009 (DCC 2009)*. IEEE Press, Mar. 2009, pp. 233–242.
3. I. Blanes and J. Serra-Sagrìstà, "Quality evaluation of progressive lossy-to-lossless remote-sensing image coding," *ICIP 2009. Proceedings of 2009 International Conference on Image Processing*, pp. 3709–3712, Nov. 2009.
4. I. Blanes and J. Serra-Sagrìstà, "Cost and scalability improvements to the Karhunen-Loève transform for remote-sensing image coding," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 48, no. 7, pp. 2854–2863, Jul. 2010.
5. L. Jimenez-Rodriguez, F. Aulí-Llinàs, J. Muñoz-Gómez, J. Bartrina-Rapesta, I. Blanes, and J. Serra-Sagrìstà, "Rate allocation method for the fast transmission of pre-encoded meteorological data over JPIP," in *In Conference on Satellite Data Compression, Communication, and Processing VI, Optical Engineering + Applications Symposium, Remote Sensing Instrumentation Program Track, SPIE Optics and Photonics*, vol. 7810, San Diego, California, Aug. 2010.
6. J. Muñoz-Gómez, J. Bartrina-Rapesta, I. Blanes, L. Jimenez-Rodriguez, F. Aulí-Llinàs, and J. Serra-Sagrìstà, "4D remote sensing image coding with JPEG2000," in *In Conference on*

- Satellite Data Compression, Communication, and Processing VI, Optical Engineering + Applications Symposium, Remote Sensing Instrumentation Program Track, SPIE Optics and Photonics*, vol. 7810, San Diego, California, Aug. 2010.
7. I. Blanes, J. Serra-Sagristà, F. Aulí-Llinàs, J. Bartrina-Rapesta, L. Jiménez-Rodríguez, and J. Muñoz-Gómez, "Side information coding for the pairwise orthogonal transform," in *In CEDI Workshop on Multimedia Data Coding and Transmission*, Valencia, Spain, Sept. 2010.
 8. F. Aulí-Llinàs, I. Blanes, J. Bartrina-Rapesta, and J. Serra-Sagristà, "Stationary model of probabilities for symbols emitted by bitplane image coders," in *In International Conference on Image Processing*. Hong Kong: IEEE Press, Sept. 2010.
 9. I. Blanes, F. García-Vílchez, and J. Serra-Sagristà, "Extensions to the CCSDS IDC recommendation for on-board hyperspectral image coding," in *2nd Int. WS on On-Board Payload Data Compression (OBPDC)*, CNES, Toulouse, France, Oct. 2010.
 10. F. García-Vílchez, J. Muñoz-Marí, M. Zorteza, I. Blanes, V. González, G. Camps-Valls, A. Plaza, and J. Serra-Sagristà, "On the impact of lossy compression on hyperspectral image classification and unmixing," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 8, no. 2, 2011.
 11. I. Blanes and J. Serra-Sagristà, "Pairwise orthogonal transform for spectral image coding," *IEEE Transactions on Geoscience and Remote Sensing*, 2010, in press.
 12. I. Blanes, J. Serra-Sagristà, and P. Schelkens, *Recent Advances in Satellite Data Compression*. Springer-Verlag, 2011, ch. Divide-and-conquer decorrelation for hyperspectral data compression.

Appendix B

Acronyms

3d-TCE	3-Dimensional Tarp-based coding with Classification for Embedding
3TERM	Triple Triangular Elementary Reversible Matrix
AE	Average Eigenvalue
AIE	Akaike Information Criterion
AKLT	Approximate Karhunen-Loève Transform
AVIRIS	Airborne Visible/Infrared Imaging Spectrometer
BIL	Band Interleaved By Line
BIP	Band Interleaved By Pixel
bpppb	bits per pixel per band
CCSDS	Consultative Committee for Space Data Systems
CCSDS-IDC	Consultative Committee for Space Data Systems recommendation for Image Data Compression
CDF	Cohen-Daubechies-Feauveau
CR	Compression-Ratio
CT	Computed Tomography
DCC	Data Compression Conference
DCT	Discrete Cosine Transform
DWT	Discrete Wavelet Transform
ED	Eigenvalue Decomposition
EIF	Empirical Indicator Function
EP	Energy Percentage
ERM	Elementary Reversible Matrix

FFT Fast Fourier Transform
GIS Geographic Information Systems
HFC Harsanyi-Farrand-Chang
ICA Independent Component Analysis
ICIP International Conference on Image Processing
IEEE Institute of Electrical and Electronics Engineers
IEM Information Extraction Measure
IWT Integer Wavelet Transform
KLT Karhunen-Loève Transform
MCT Multi-component Transform
MDL Minimum Description Length
MRI Magnetic Resonance Imaging
MSA Maximum Spectral Angle
MSE Mean Squared Error
MSS Maximum Spectral Similarity
PA Parallel Analysis
PCRD Post-Compression Rate-Distortion
PLL Progressive Lossy-to-Lossless
POC Preservation of Classification
POT Pairwise Orthogonal Transform
PRNG Pseudo-Random Number Generator
R-D Rate-Distortion
RKLT Reversible Karhunen-Loève Transform
RS Remote Sensing
RX Reed Xiaoli
SERM Single-row Elementary Reversible Matrix
SNR Signal-to-Noise Ratio
TCE Tarp-based coding with Classification for Embedding
TERM Triangular Elementary Reversible Matrix
ERMs Elementary Reversible Matrices
TERMs Triangular Elementary Reversible Matrices
SERMs Single-row Elementary Reversible Matrices
MCTs Multicomponent Transforms

Appendix C

Implementation particularities

This appendix intends to be a short complementary reference into the technical details of an implementation of divide-and-conquer transforms. Its main objective is to highlight the set of issues that are usually deemed not relevant for scientific publications but that might be of interest when the theory is put into practice. In addition directions are given into the best procedures for each stage of a transform by pointing at the best references of each procedure.

OVERVIEW AND CONTROL LOGIC

A divide-and-conquer transform is based on different stages. An overview of such stages is shown in Fig. C.1. Stages can be clearly classified according to whether they function as control elements organizing the operation of local transforms, or whether they are part of local transforms and directly manipulate the information to produce the final result.

Control elements are represented in Fig. C.1 by three different stages according to their function in the information flow. They are a highly interconnected group of stages that has to be planned as a whole element. It is a function of the control elements that the transform can be applied using a minimal amount of memory, by organizing the operations such that some of the portions of the output can be produced earlier. In Fig. C.2, a diagram shows an example of an eight-component POT where components are read sequentially, and where outputs are being stored just after they are obtained. In said example, an efficient allocation strategy requires a logarithmic amount of memory in relation to the total amount of components.

A possible implementation of such control logic is the use of a consumer/producer model in which different class instances represent each of the transform clusters, and they forward components between them as they have them available. In Fig. C.3, it is shown a class diagram for a consumer/producer model based on the one used in the fast implementation of the POT [43].

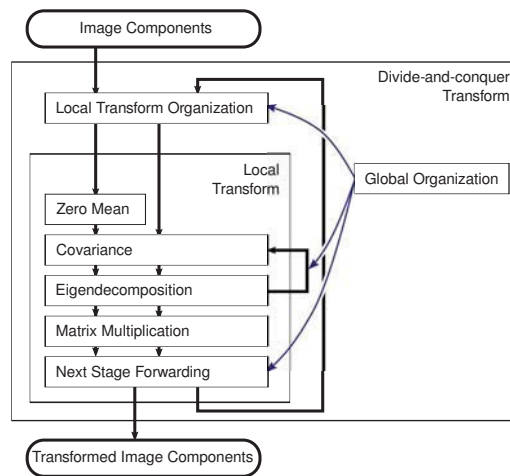


Figure C.1: Overview diagram of a divide-and-conquer transform.

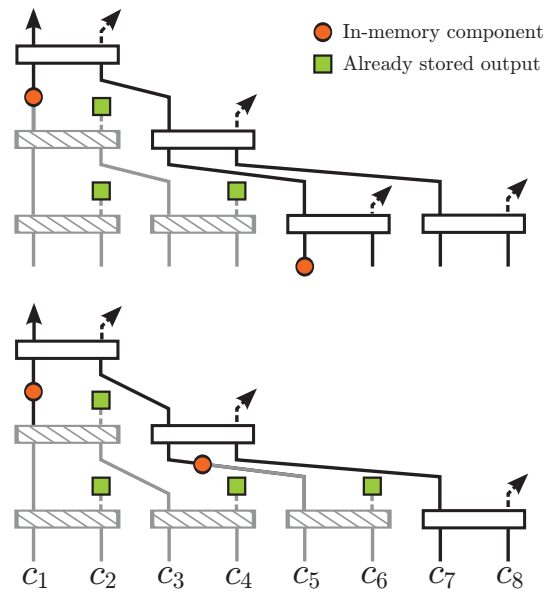


Figure C.2: Internal state of an eight component transform when components are read sequentially. Internal state before (top), and after (bottom), the sixth component of the image (c_6) is read. Figure is from [32].

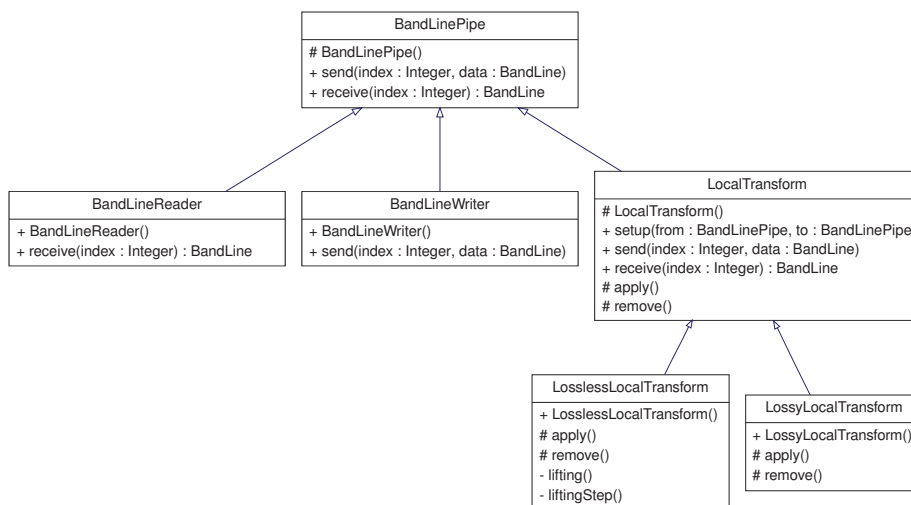


Figure C.3: Class diagram of a consumer/producer model for multilevel transforms.

COVARIANCE MATRIX AND SUBSAMPLING

In case components do not have zero mean, a stage to offset them is prepended to the covariance matrix calculation. This stage simply modifies a component, say x , by subtracting the current mean of the component to it, i.e., $x'_i = x_i - \bar{x}$, or $x'_i = x_i - \lceil \bar{x} \rceil$ in the case of a lossless transform.

Therefore, as components have zero mean due to a stage with this purpose, it is often safe to assume that the covariance of two components can be computed by

$$\sigma(x', y') = \sum_{i=0}^m x'_i y'_i. \quad (\text{C.1})$$

For images where a lossless zero-mean stage has been applied, the previous formula can still be used, except, for high bitdepth images. The unrounded mean from the zero-mean stage is required on images with a bitdepth well over 16 bpppb where a lossless transform is being applied. The traditional formula can be used, i.e.,

$$\sigma(x', y') = \sum_{i=0}^m (x'_i - \overline{x'}) (y'_i - \overline{y'}), \quad (\text{C.2})$$

where the unrounded mean of the zero-mean images is available from the zero-mean stage as $\overline{x'} = \bar{x} - \lceil \bar{x} \rceil$.

☞ **NOTE:** Beware of using the “fast” covariance formula, i.e.,

$$\sigma(x', y') = \sum_{i=0}^m (x'_i y'_i - \overline{x'} \overline{y'}), \quad (\text{C.3})$$

because it produces a catastrophic cancellation due to the subtraction of two big similar values [47], such as $a - b = 0$, where $a = 2^{30} + 1$ and $b = 2^{30}$, due to a lack of precision in the variable a . See [48] for a detailed report on the stable computation of covariances.

The subsampling covariance calculation optimization is introduced and well explained in [7]. Experimental results show that the factor of subsampling is dependent on both the coding bitdepth and the spatial region size, but not on the quality of the sampling randomness. A Park-Miller Pseudo-Random Number Generator (PRNG) is often well suited for such purpose.

$$\begin{aligned} Y_n &= M \cdot X_n \operatorname{div} (2^{32} - 5) \\ X_n &= (279470273 \cdot X_{n-1}) \operatorname{mod} (2^{32} - 5) \end{aligned} \quad (\text{C.4})$$

In addition, a further optimization involves the knowledge that the eigenvalues of the eigendecomposition of a KLT correspond to the variances of the output components of the KLT. In other words, the variance of an output component is the associated λ_i value of the Λ diagonal matrix.

$$\Sigma_Y = (1/M)YY^T = (1/M)Q^T XX^T Q = Q^T \Sigma_X Q = \Lambda$$

Using this knowledge, the diagonal values of the covariance matrix of transform levels beyond one can be obtained from previous transform levels and do not need to be calculated.

EIGENDECOMPOSITION

The eigendecomposition stage is usually one of the trickiest stages. It does not have a significant computational cost, nor memory need, yet it is easy to omit some floating point issues that might produce very incorrect results, such as a catastrophic cancellation.

Algorithms are usually based on iterative QR decompositions, after a tridiagonalization by Householder transformations as a preprocessing stage. A good, and often cited, reference of the procedure can be found in [49]. Although it is certainly possible to implement this stage, it is heavily recommended to use an existing library containing an eigendecomposition procedure. A common library is the GNU Scientific Library (GSL) [50], which implements the iterative QR decomposition procedure as described above.

LOSSLESS

For lossless application, the lifting factorization introduced in [27] is usually used, with the variation of quasi-complete pivoting [28]. Although the procedure is quite complicated, it is usually straightforward to implement in practice and obtain the sequence of lifting steps equivalent to the lossy transform.

The key issue of this procedure is to ensure reversibility of the lifting steps. Lifting steps contain a sequence of floating-point or fixed-point operations that will be rounded to an integer. It is critical that these sequence of operations are performed in the very same order both on the forward and inverse transforms, and with the same bit-exact result. A fixed-point implementation of these procedures might be the best solution, as standardized in [51], although, a java implementation might also guarantee bit-exact results with the use of the `strictfp` keyword.

VALUE SATURATION

Another issue appears during the inverse transform application on a lossy-coded image. It cannot be assumed that the bitdepth of an image is preserved when lossy coding is applied, or in other words, the bitdepth of an image may, and often does, grow due to the distortion on a lossy image coder. When overflows are not taken into account, it is easy to see, for example, a lossy coding of a signed 16 bpppb image with a higher than usual MSE, due to some values overflowing and becoming negative. One has to be careful to check for overflows that might occur due to this phenomenon, and saturate the overflowing values to the nearest valid result, which, in fact, will almost always be a better result than if the output precision was extended to handle larger values (because these larger values were certainly smaller on the original image).

Bibliography

- [1] D. E. Knuth, *Sorting and Searching*, 2nd ed., ser. The Art of Computer Programming. Addison-Wesley, 1998, vol. 3.
- [2] K. Karhunen, "Zur spektraltheorie stochastischer prozesse," *Ann. Acad. Sci. Fennicae*, 1946.
- [3] M. Loève, "Fonctions aleatoires de seconde ordre," in *Processus Stochastiques et Mouvement Brownien*, P. Lévy, Ed. Paris: Gauthier-Villars, 1948.
- [4] K. Pearson, "On lines and planes of closest fit to systems of points in space," *Philosophical Magazine*, vol. 2, no. 6, pp. 559–572, 1901.
- [5] H. Hotelling, "Analysis of a complex of statistical variables into principal components," *Journal of Educational Psychology*, vol. 24, no. 6, pp. 417–441, Sept. 1933.
- [6] N. Ahmed, T. Natarajan, and K. Rao, "Discrete cosine transform," *IEEE Transactions on Computers*, vol. C-23, no. 1, pp. 90–93, Jan. 1974.
- [7] B. Penna, T. Tillo, E. Magli, and G. Olmo, "Transform coding techniques for lossy hyperspectral data compression," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 45, no. 5, pp. 1408–1421, May 2007.
- [8] L. Lan and I. Reed, "Fast approximate Karhunen-Loeve transform (AKLT) with applications to digital image-coding," *Elsevier Visual Commun. and Image Process.* 93, vol. 2094, pp. 444–455, 1993.
- [9] A. Pirooz and I. Reed, "A new approximate Karhunen-Loeve transform for data compression," *Conference Record of the Thirty-Second Asilomar Conference on Signals, Systems and Computers*, vol. 1-2, pp. 1471–1475, 1998.
- [10] I. R. Greenshields and J. A. Rosiene, "A fast wavelet-based Karhunen-Loeve transform," *Pattern Recognition*, vol. 31, no. 7, pp. 839–845, Jul. 1998.
- [11] M. Cagnazzo, L. Cicala, G. Poggi, and L. Verdoliva, "Low-complexity compression of multispectral images based on classified transform coding," *Elsevier Signal Processing-Image Communication*, vol. 21, no. 10, pp. 850–861, 2006.

- [12] C. Thiebaut, E. Christophe, D. Lebedeff, and C. Latry, "CNES studies of on-board compression for multispectral and hyperspectral images," in *Satellite Data Compression, Communications, and Archiving III*, R. W. Heymann, B. Huang, and I. Gladkova, Eds., vol. 6683, no. 1. SPIE, 2007, p. 668305.
- [13] L. Wang, J. Wu, L. Jiao, and G. Shi, "3D medical image compression based on multiplierless low-complexity RKLT and shape-adaptive wavelet transform," *ICIP 2009. Proceedings of 2009 International Conference on Image Processing*, pp. 2521–2524, Nov. 2009.
- [14] —, "Lossy-to-lossless hyperspectral image compression based on multiplierless reversible integer TDLT/KLT," *IEEE Geoscience and Remote Sensing Letters*, vol. 6, no. 3, pp. 587–591, July 2009.
- [15] I. P. Akam Bitá, M. Barret, and D.-T. Pham, "On optimal orthogonal transforms at high bit-rates using only second order statistics in multicomponent image coding with JPEG2000," *Elsevier Signal Processing*, vol. 90, no. 3, pp. 753 – 758, 2010.
- [16] —, "On optimal transforms in lossy compression of multicomponent images with JPEG2000," *Elsevier Signal Processing*, vol. 90, no. 3, pp. 759–773, 2010.
- [17] M. Barret, J.-L. Gutzwiller, I. P. Akam Bitá, and F. Dalla Vedova, "Lossy hyperspectral images coding with exogenous quasi optimal transforms," in *Data Compression Conf. 2009 (DCC 2009)*. IEEE Press, Mar. 2009, pp. 411–419.
- [18] I. P. Akam Bitá, M. Barret, F. Dalla Vedova, and J.-L. Gutzwiller, "Lossy compression of MERIS superspectral images with exogenous quasi optimal coding transforms," in *Satellite Data Compression, Communication, and Processing V*, B. Huang, A. J. Plaza, and R. Vitulli, Eds., vol. 7455. SPIE, 2009, p. 74550U.
- [19] M. Gastpar, P. Dragotti, and M. Vetterli, "The distributed Karhunen-Loeve transform," vol. 52, no. 12, pp. 5177–5196, 2006.
- [20] Y. Wongsawat, S. Oraintara, and K. R. Rao, "Integer sub-optimal Karhunen-Loève transform for multi-channel lossless EEG compression," *European Signal Processing Conference*, 2006.
- [21] Y. Wongsawat, "Lossless compression for 3-D MRI data using reversible KLT," *Int'l Conf. on Audio, Language and Image Processing, 2008. (ICALIP 2008).*, pp. 1560–1564, July 2008.
- [22] Q. Du, W. Zhu, H. Yang, and J. E. Fowler, "Segmented principal component analysis for parallel compression of hyperspectral imagery," *IEEE Geoscience and Remote Sensing Letters*, vol. 6, no. 4, pp. 713–717, Oct. 2009.
- [23] J. A. Saghri and S. Schroeder, "An adaptive two-stage KLT scheme for spectral decorrelation in hyperspectral bandwidth compression," *Proc. SPIE*, vol. 7443, p. 744313, Sept. 2009.

- [24] J. A. Saghri, S. Schroeder, and A. G. Tescher, "Adaptive two-stage Karhunen-Loeve-transform scheme for spectral decorrelation in hyperspectral bandwidth compression," *SPIE Optical Engineering*, vol. 49, p. 057001, May 2010.
- [25] I. Blanes, J. Serra-Sagrìstà, and P. Schelkens, *Recent Advances in Satellite Data Compression*. Springer-Verlag, 2011, ch. Divide-and-conquer decorrelation for hyperspectral data compression.
- [26] I. Blanes and J. Serra-Sagrìstà, "Clustered reversible-KLT for progressive lossy-to-lossless 3d image coding," in *Data Compression Conf. 2009 (DCC 2009)*. IEEE Press, Mar. 2009, pp. 233–242.
- [27] P. W. Hao and Q. Y. Shi, "Matrix factorizations for reversible integer mapping," *IEEE Transactions on Signal Processing*, vol. 49, no. 10, pp. 2314–2324, 2001.
- [28] L. Galli and S. Salzo, "Lossless hyperspectral compression using KLT," *IEEE Int'l Geosci. and Remote Sens. Symp. Proc. (IGARSS 2004)*, vol. 1-7, pp. 313–316, 2004.
- [29] D. Taubman and M. Marcellin, *JPEG2000: Image Compression Fundamentals, Standards, and Practice*. Kluwer International Series in Engineering and Computer Science, 2002, vol. 642.
- [30] P. Schelkens, A. Skodras, and T. Ebrahimi, Eds., *The JPEG 2000 Suite*. Wiley, Sept. 2009.
- [31] I. Blanes and J. Serra-Sagrìstà, "Cost and scalability improvements to the Karhunen-Loève transform for remote-sensing image coding," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 48, no. 7, pp. 2854–2863, Jul. 2010.
- [32] ———, "Pairwise orthogonal transform for spectral image coding," *IEEE Transactions on Geoscience and Remote Sensing*, 2010, in press.
- [33] E. Christophe, D. Leger, and C. Mailhes, "Quality criteria benchmark for hyperspectral imagery," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 43, no. 9, pp. 2103–2114, Sept. 2005.
- [34] S. Rupert, M. Sharp, J. Sweet, and E. Cincotta, "Noise constrained hyperspectral data compression," *Int'l Geosci. Remote Sens. Symp. 2001. (IGARSS 01)*, vol. 1, pp. 94–96 vol.1, Jul. 2001.
- [35] Z. Wang and A. Bovik, "A universal image quality index," *IEEE Signal Processing Letters*, vol. 9, no. 3, pp. 81–84, Mar. 2002.
- [36] I. Blanes and J. Serra-Sagrìstà, "Quality evaluation of progressive lossy-to-lossless remote-sensing image coding," *ICIP 2009. Proceedings of 2009 International Conference on Image Processing*, pp. 3709–3712, Nov. 2009.
- [37] J. B. MacQueen, "Some methods for classification and analysis of multivariate observations," in *Proc. 5th Berkeley Symp. Math. Statistics and Probability*, L. M. L. Cam and J. Neyman, Eds., vol. 1. University of California Press, 1967, pp. 281–297.

- [38] I. Reed and X. Yu, "Adaptive multiple-band CFAR detection of an optical pattern with unknown spectral distribution," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 38, no. 10, pp. 1760–1770, Oct. 1990.
- [39] Group on Interactive Coding of Images, "Spectral transform software," <http://gici.uab.cat/>, 2010.
- [40] B. Penna, T. Tillo, E. Magli, and G. Olmo, "A new low complexity KLT for lossy hyperspectral data compression," *IGARSS 2006. IEEE International Conference on Geoscience and Remote Sensing Symposium, 2006.*, pp. 3525–3528, 2006.
- [41] Group on Interactive Coding of Images, "iometrics software," <http://gici.uab.cat/>, 2010.
- [42] A. Eskicioglu and P. Fisher, "Image quality measures and their performance," *IEEE Transactions on Communications*, vol. 43, no. 12, pp. 2959–2965, Dec 1995.
- [43] Group on Interactive Coding of Images, "The pairwise orthogonal transform software," <http://gici.uab.cat/>, 2010.
- [44] D. Taubman, "Kakadu software," <http://www.kakadusoftware.com/>, 2000.
- [45] Jet Propulsion Laboratory, NASA, "Hyperspectral image compression website," <http://compression.jpl.nasa.gov/hyperspectral/>.
- [46] A. Cohen, I. Daubechies, and J. Feauveau, "Biorthogonal basis of compactly supported wavelets," *Comm. Pure Appl. Math.*, vol. 45, pp. 485–560, 1992.
- [47] D. E. Knuth, *Seminumerical Algorithms*, 2nd ed., ser. The Art of Computer Programming. Addison-Wesley, 1998, vol. 2.
- [48] P. Pébay, "Formulas for robust, one-pass parallel computation of covariances and arbitrary-order statistical moments," Sandia National Laboratories, Tech. Rep. SAND2008-6212, 2008.
- [49] G. Golub and C. van Loan, *Matrix Computations*. The Johns Hopkins University Press, Oct. 1996.
- [50] B. Gough, Ed., *GNU Scientific Library Reference Manual*, 3rd ed. Network Theory Ltd., Jan. 2009.
- [51] "JPEG 2000 image coding system - Part 2: Extensions, Information technology 15444-2. International Organization for Standardization / International Electrotechnical Commission (ISO/IEC); International Telecommunication Union-Telecom Standardization (ITU-T)," International Organization for Standardization / International Electrotechnical Commission (ISO/IEC); International Telecommunication Union-Telecom Standardization (ITU-T), May 2004.

- [52] J. Yang, Y. Wang, W. Xu, and Q. Dai, "Image coding using dual-tree discrete wavelet transform," *IEEE Transactions on Image Processing*, vol. 17, no. 9, pp. 1555–1569, Sept. 2008.
- [53] J. Zhang, J. E. Fowler, and G. Liu, "Lossy-to-lossless compression of hyperspectral imagery using three-dimensional TCE and an integer KLT," *IEEE Geoscience and Remote Sensing Letters*, vol. 5, pp. 814–818, Oct. 2008.
- [54] F. A. M. L. Bruekers and A. W. M. van den Eenden, "New networks for perfect inversion and perfect reconstruction," *IEEE Journal on Selected Areas in Communications*, vol. 10, no. 1, pp. 130–137, 1992.
- [55] P. Hao and Q. Shi, "Reversible integer KLT for progressive-to-lossless compression of multiple component images," *ICIP 2003. Proceedings of 2003 International Conference on Image Processing*, vol. 1, pp. 633–636, 2003.
- [56] Y. She, P. Hao, and Y. Paker, "Matrix factorizations for parallel integer transforms," *Parallel Architectures, Algorithms and Networks, 2004. Proceedings. 7th International Symposium on*, pp. 254–259, May 2004.
- [57] P. van Vugt and M. Adams, "Reversible integer-to-integer wavelet transforms with improved approximation properties," *39th Asilomar Conference on Signals, Systems and Computers*, vol. 1-2, pp. 247–251, 2005.
- [58] S. Pei and J. Ding, "Improved reversible integer transform," *Proceedings of 2006 IEEE International Symposium on Circuits and Systems*, vol. 1-11, pp. 1091–1094, 2006.
- [59] X. Luo, L. Guo, and Z. Liu, "Lossless compression of hyperspectral imagery using integer principal component transform and 3-D Tarp coder," *SNPD 2007. Proceedings of Eighth Acis International Conference on Software Engineering, Artificial Intelligence, Networking, and Parallel/Distributed Computing*, vol. 1, pp. 553–558, 2007.
- [60] J. Serra-Sagristà and F. Aulí-Llinàs, *Computational Intelligence for Remote Sensing*, ser. Studies in Computational Intelligence. Springer, Jul. 2008, vol. 133, ch. Remote Sensing Data Compression, pp. 27–61.
- [61] R. Leung and D. S. Taubman, "Transform and embedded coding techniques for maximum efficiency and random accessibility in 3-d scalable compression," *IEEE Transactions on Image Processing*, vol. 14, no. 10, pp. 1632–1646, Oct. 2005.
- [62] A. Said and W. Pearlman, "An image multiresolution representation for lossless and lossy compression," *IEEE Transactions on Image Processing*, vol. 5, no. 9, pp. 1303–1310, Sep. 1996.
- [63] G. Ginesu, D. Giusto, and W. Pearlman, "Lossy to lossless SPIHT-based volumetric image compression," *Proc. Int'l Conf. Acoust., Speech, Signal Process., 2004. (ICASSP 04).*, vol. 3, pp. iii–693–6 vol.3, May 2004.

- [64] R. D. Maesschalck, D. Jouan-Rimbaud, and D. L. Massart, "The mahalanobis distance," *Chemometrics and Intelligent Laboratory Systems*, vol. 50, no. 1, pp. 1 – 18, 2000. [Online]. Available: <http://www.sciencedirect.com/science/article/B6TFP-3Y8VGYK-1/2/92ac3e8ac922320df9a62b096aca9bee>
- [65] Jet Propulsion Laboratory, NASA, "Airborne Visible InfraRed Imaging Spectrometer website," <http://aviris.jpl.nasa.gov/html/aviris.overview.html>.
- [66] E. Magli, G. Olmo, and E. Quacchio, "Optimized onboard lossless and near-lossless compression of hyperspectral data using calic," *IEEE Geoscience and Remote Sensing Letters*, vol. 1, no. 1, pp. 21–25, Jan. 2004.
- [67] J. Mielikainen, "Lossless compression of hyperspectral images using lookup tables," *IEEE Signal Processing Letters*, vol. 13, no. 3, pp. 157–160, Mar. 2006.
- [68] A. Abrardo, M. Barni, E. Magli, and F. Nencini, "Error-resilient and low-complexity onboard lossless compression of hyperspectral images by means of distributed source coding," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 48, no. 4, pp. 1892–1904, Apr. 2010.
- [69] R. B. Cattell, "The scree test for the number of factors," *Multivariate Behav. Res.*, vol. 1, pp. 245–76, 1966.
- [70] E. R. Malinowski, "Determination of the number of factors and the experimental error in a data matrix," *Analytical Chemistry*, vol. 49, no. 4, pp. 612–617, 1977.
- [71] J. L. Horn, "A rationale and a test for the number of factors in factor analysis," *Psychometrika*, vol. 30, pp. 179–185, 1965.
- [72] H. Akaike, "A new look at the statistical model identification," *IEEE Transactions on Automatic Control*, vol. 19, no. 6, pp. 716–723, Dec 1974.
- [73] G. Schwarz, "Estimating the dimension of a model," *Annals of Statistics*, vol. 6, pp. 461–464, 1978.
- [74] J. Rissanen, "Modeling by shortest data description," *Automatica*, vol. 14, pp. 465–471, 1978.
- [75] J. C. Harsanyi, W. Farrand, and C.-I. Chang, "Determining the number and identity of spectral endmembers: An integrated approach using Neyman-Pearson eigenthresholding and iterative constrained RMS error minimization," *Proc. of the 9th Thematic Conf. on Geologic Remote Sens.*, pp. 395–408, Feb. 1993.
- [76] U.S. Geological Survey and NASA, "Earth Observing 1, Hyperion website," <http://eo1.usgs.gov/hyperion.php>.
- [77] F. Garcia-Vilchez and J. Serra-Sagrìstà, "Extending the CCSDS Recommendation for Image Data Compression for Remote Sensing Scenarios," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 47, no. 10, pp. 3431 –3445, 2009.

- [78] Consultative Committee for Space Data Systems, "Image Data Compression," Blue Book, CCSDS 122.0-B-1, Nov. 2005.
- [79] J. Fowler, "QccPack: An open-source software library for quantization, compression, and coding," in *Applications of Digital Image Processing XXIII*, A. Tescher, Ed., vol. 4115, San Diego, CA, USA, Aug. 2000, pp. 294–301.
- [80] Group on Interactive Coding of Images, "TER software; Open Source CCSDS-122-B-1 implementation and extension." <http://www.gici.uab.cat/TER>, <http://ter.sourceforge.net/>, June 2008.
- [81] A. Lucero, S. Cabrera, E. J. Vidal, and A. Aguirre, "Evaluating residual coding with JPEG2000 for L-infinity driven hyperspectral image compression," *Satellite Data Compression, Communications, and Archiving*, vol. 5889, no. 1, pp. 12–23, 2005. [Online]. Available: <http://link.aip.org/link/?PSI/5889/588903/1>
- [82] G. Carvajal, B. Penna, and E. Magli, "Unified lossy and near-lossless hyperspectral image compression based on JPEG 2000," *IEEE Geoscience and Remote Sensing Letters*, vol. 5, no. 4, pp. 593–597, Oct. 2008.
- [83] B. Penna, T. Tillo, E. Magli, and G. Olmo, "Hyperspectral image compression employing a model of anomalous pixels," *IEEE Geoscience and Remote Sensing Letters*, vol. 4, no. 4, pp. 664–668, 2007.
- [84] M. Klimesh, "Low-complexity adaptive lossless compression of hyperspectral imagery," *Proc. SPIE*, pp. 63 000N.1–63 000N.9, Sept. 2006.
- [85] J. Mielikainen and P. Toivanen, "Lossless compression of hyperspectral images using a quantized index to lookup tables," *IEEE Geoscience and Remote Sensing Letters*, vol. 5, no. 3, pp. 474–478, July 2008.
- [86] A. Kiely and M. Klimesh, "Exploiting calibration-induced artifacts in lossless compression of hyperspectral imagery," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 47, no. 8, pp. 2672–2678, Aug. 2009.
- [87] J. Pearlman, P. Barry, C. Segal, J. Shepanski, D. Beiso, and S. Carman, "Hyperion, a space-based imaging spectrometer," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 41, no. 6, pp. 1160–1173, Jun. 2003.
- [88] F. Kruse, J. Boardman, and J. Huntington, "Comparison of airborne hyperspectral data and EO-1 Hyperion for mineral mapping," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 41, no. 6, pp. 1388–1400, June 2003.
- [89] U.S. Geological Survey, "Earth explorer," <http://earthexplorer.usgs.gov>.

- [90] P. Deutsch and J.-L. Gailly, "ZLIB compressed data format specification version," May 1996. [Online]. Available: <http://www.gzip.org/zlib/rfc1950.pdf>
- [91] B. Penna, T. Tillo, E. Magli, and G. Olmo, "Progressive 3-D coding of hyperspectral images based on JPEG2000," *IEEE Geoscience and Remote Sensing Letters*, vol. 3, no. 1, pp. 125–129, Jan. 2006.
- [92] J. E. Fowler and J. T. Rucker, *Hyperspectral Data Exploitation: Theory and Applications*. Hoboken, NJ, USA: John Wiley & Sons Inc., 2007, ch. "3D wavelet-Based Compression of Hyperspectral Imager", pp. 379–407.
- [93] E. Magli, "Multiband lossless compression of hyperspectral images," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 47, no. 4, pp. 1168–1178, Apr. 2009.
- [94] A. Abrardo, M. Barni, E. Magli, and F. Nencini, "Error-resilient and low-complexity onboard lossless compression of hyperspectral images by means of distributed source coding," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 48, no. 4, pp. 1892–1904, Apr. 2010.
- [95] C.-I. Chang and S.-S. Chiang, "Anomaly detection and classification for hyperspectral imagery," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 40, no. 6, pp. 1314–1325, Jun. 2002.

